# Constraint in Automated HBT Method

By Bohdan Raczynski, August 3023

In Part I, titled “Automated Method For Minimum-Phase Extraction Using HBT”, the concept of constrained optimization was introduced. In this paper, we will examine this aspect in more details.

We now return to the tweeter driver presented in Part I.

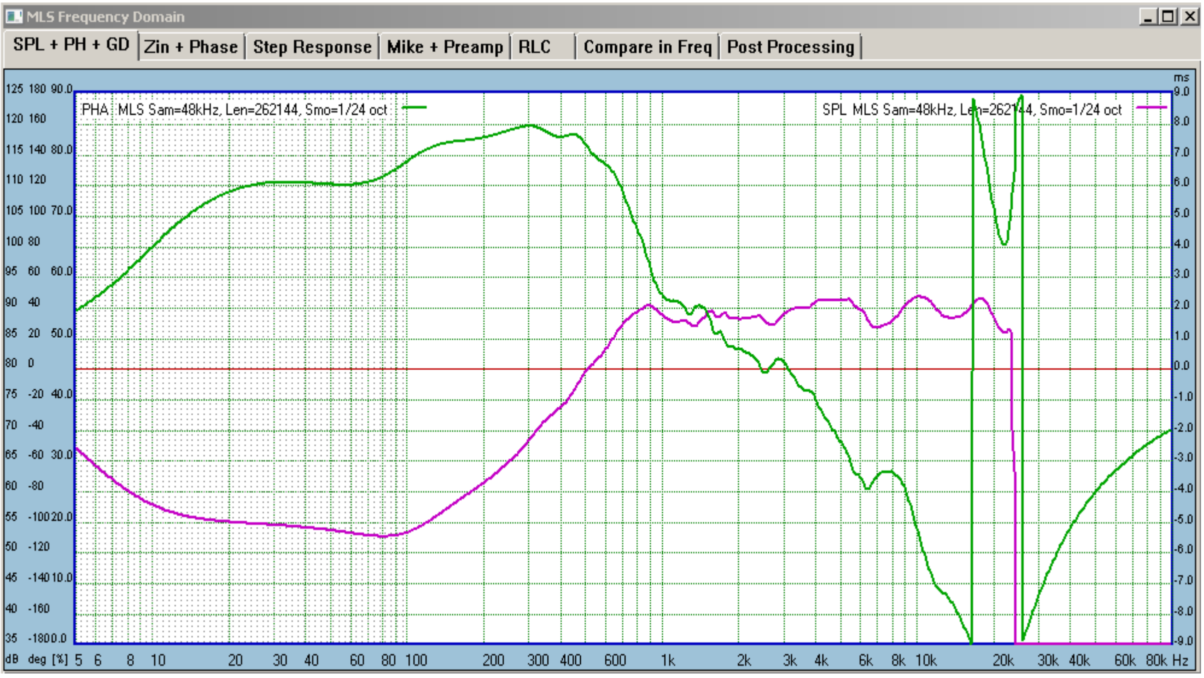



Figure 1. Example of SPL/Phase response of a tweeter driver.

Please note, that there is virtually no SPL/Phase data above 22kHz. Still, we should run the minimum-phase extraction over several possible attachment points and examine the results.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Driver's Name | | Jaycar_DT25_25cm_dist.res | | | | |
| | | | | | | | | |
| Start Bin = | 85 | Smooth = | 1/24dB/oct | | Window = | 10ms | | |
| Start HBT | 400Hz | 12dB | 22000Hz | 40dB | | | | |
| | | | | | | | | |
| HBT LP Start | HBT HP Start | FFT Bin | Delay ms | LP Order | HP Order | Error | Bins | Error |
| 17000 | 400 | 90 | 0.002 | 8.94 | 11.99 | 330.558 | 284 | 1.16394 |
| 18000 | 400 | 90 | -0.002 | 14.38 | 12 | 366.937 | 289 | 1.26968 |
| 19000 | 400 | 90 | -0.002 | 14.75 | 11.99 | 398.982 | 293 | 1.36171 |
| 20000 | 400 | 90 | 0 | 11.93 | 12 | 422.975 | 297 | 1.42416 |
| 21000 | 400 | 90 | 0.002 | 8.97 | 12 | 464.848 | 300 | 1.54949 |
| 22000 | 400 | 90 | 0 | 10.85 | 11.99 | 630.953 | 304 | 2.0755 |
| 23000 | 400 | 90 | -0.004 | 16.44 | 11.97 | 759.751 | 307 | 2.47476 |

Frequency responses of Tweeter 1 shown on Figure 1, that frequency range 17-20kHz will meaningfully contribute to the accuracy of the process, and 22kHz frequency may be too close to the limit of reliable data. Therefore, 21kHz attachment point is selected. It is observable, that low-pass slope attached at 21000Hz is not doing its job properly. Resulting phase response is too shallow. See Figure 2 below.

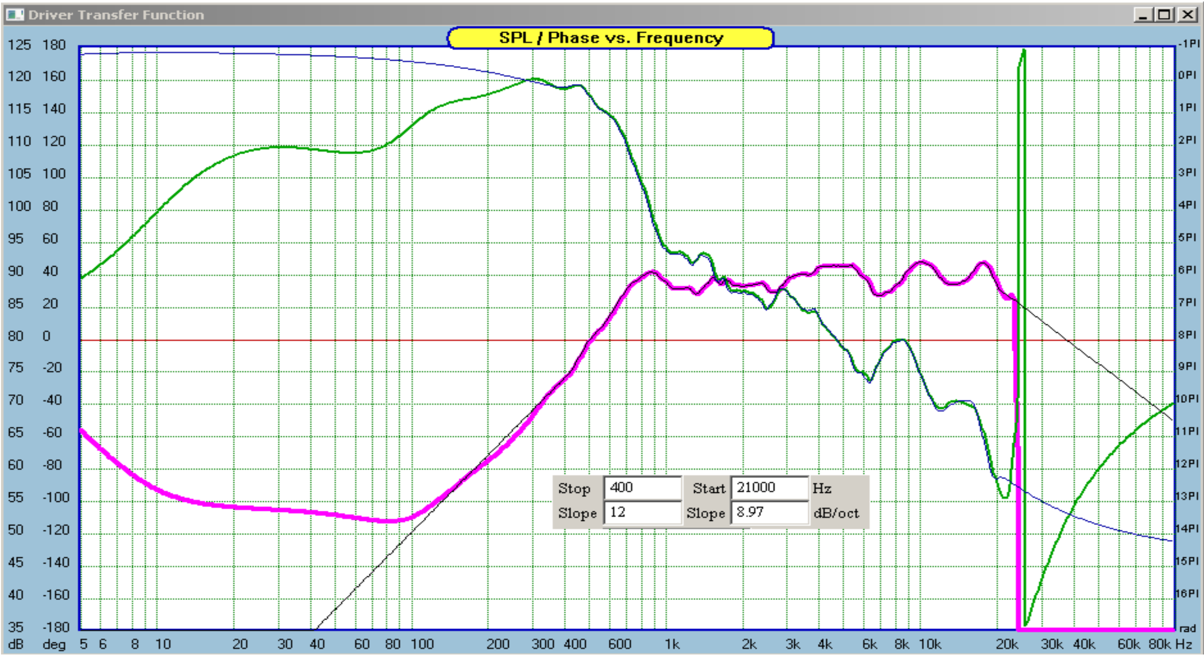


Figure 2. Shallow phase response.

Constrained IHBT optimization at 21000Hz leads to the following result.

| IHBT LP Start | IHBT HP Start | FFT Bin | Delay ms | LP Order | HP Order | Error | Bins | IHBT Error | Normalized Error |
|---|---|---|---|---|---|---|---|---|---|
| 17000 | 400 | 90 | -0.008 | 18.06 | 11.91 | 670.741 | 284 | 9.95 | 2.36176 |
| 18000 | 400 | 90 | -0.008 | 20.2 | 11.94 | 486.404 | 289 | 10.07 | 1.68306 |
| 19000 | 400 | 90 | -0.01 | 22.94 | 11.94 | 684.873 | 293 | 11.17 | 2.33745 |
| 20000 | 400 | 90 | -0.01 | 22.72 | 11.91 | 979.844 | 297 | 14.25 | 3.29914 |
| 21000 | 400 | 89 | 0.01 | 23.43 | 12.02 | 1243.452 | 300 | 18.88 | 4.14484 |
| 22000 | 400 | 89 | 0.01 | 23.69 | 12.01 | 1185.611 | 304 | 21.66 | 3.90004 |
| 23000 | 400 | 90 | -0.006 | 18.9 | 11.95 | 759.968 | 307 | 19.45 | 2.47547 |

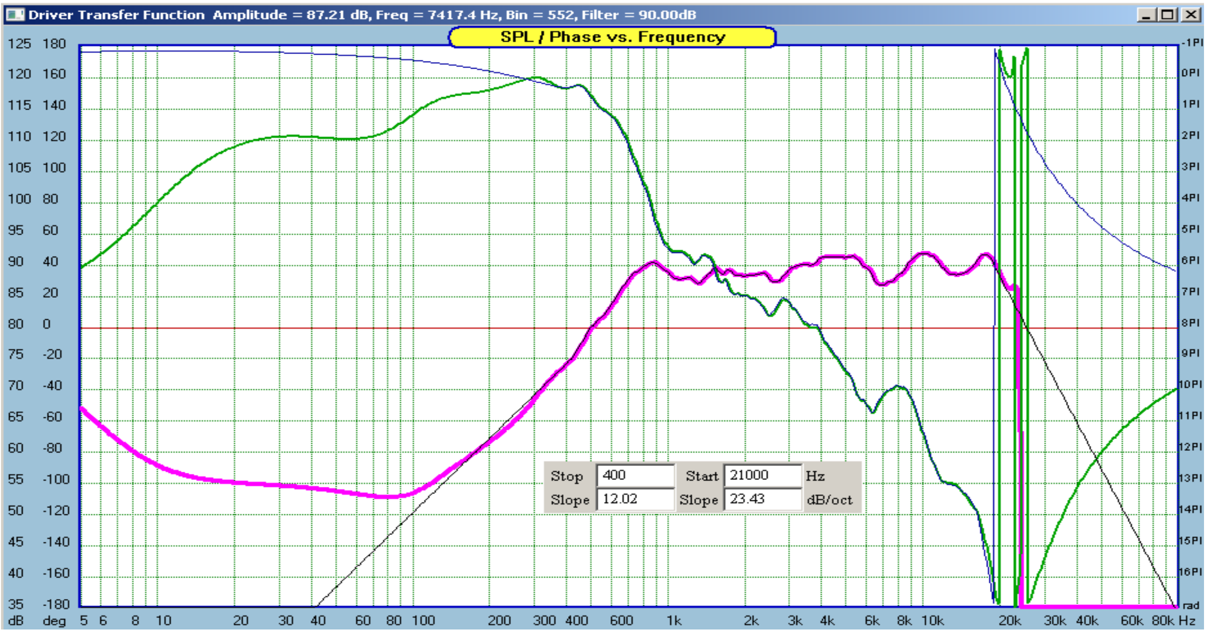


Figure 3. Phase response obtained using constrained optimization.

Next driver is a woofer driver.

Examination of the frequency response would indicate problematic SPL response above 5kHz.

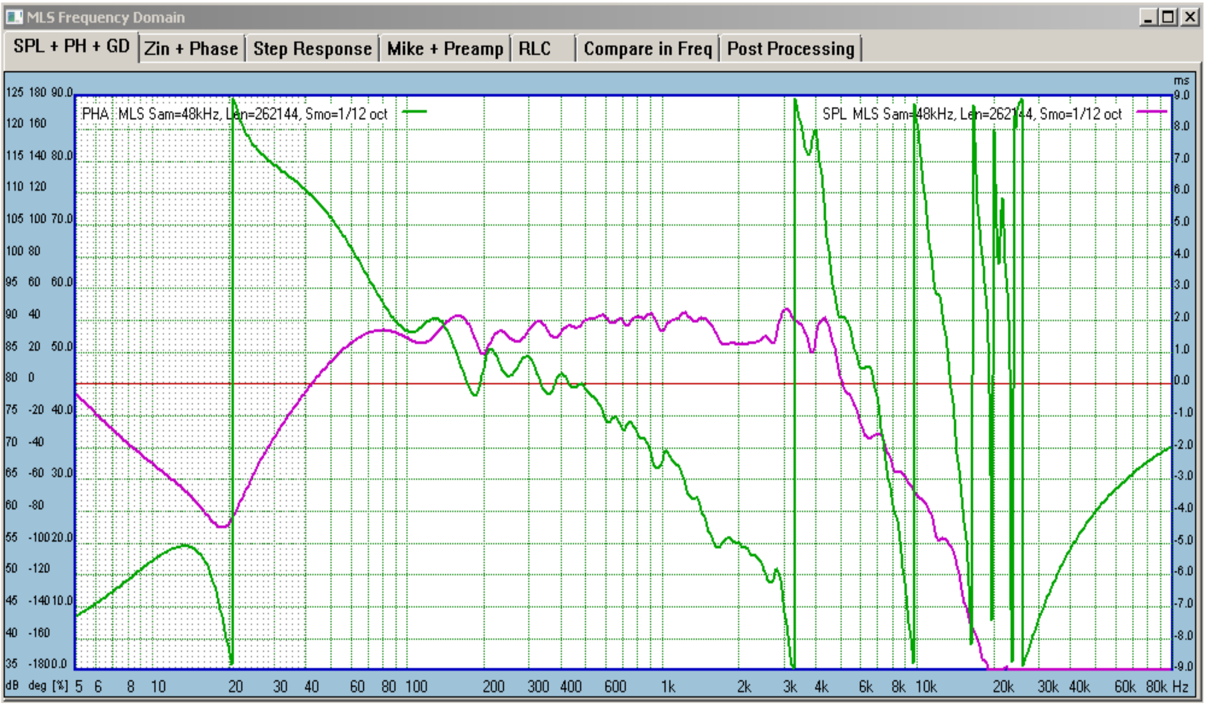



Figure 4. Example of SPL/Phase response of a woofer driver

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Driver's Name | | Rear_Speaker_50cm.res | | | | |
| HBT Method | | | | | | | | |
| Start Bin = | 119 | Smooth = | 1/12 dB/oct | | Window = | 25ms | | |
| Start HBT | 50Hz | 18.5 | 4000 | 40 | | | | |
| | | | | | | After HBT | | Normalized |
| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | Error |
| 4000 | 50 | 127 | 0.008 | 12.5 | 9.96 | 486.676 | 332 | 1.46589 |
| 5000 | 50 | 123 | 0.008 | 38.42 | 9.92 | 385.908 | 349 | 1.10575 |
| 6000 | 50 | 124 | 0 | 36.08 | 9.95 | 542.306 | 362 | 1.49808 |
| 7000 | 50 | 124 | -0.006 | 42.36 | 9.96 | 718.29 | 374 | 1.92056 |
| 8000 | 50 | 122 | 0.004 | 60.48 | 9.95 | 635.101 | 384 | 1.65391 |
| 9000 | 50 | 120 | 0.002 | 88.43 | 9.92 | 586.1 | 393 | 1.49135 |
| 10000 | 50 | 117 | 0.006 | 129.45 | 9.9 | 770.884 | 401 | 1.9224 |

Unconstrained HBT optimization is shown on Figure 5 below. If it wasn't for the tabulated Errors, it would be difficult to visually determine the best phase match. Even so, the Normalized Errors are not far from each other between 6000Hz attachment point, and 10000Hz result..

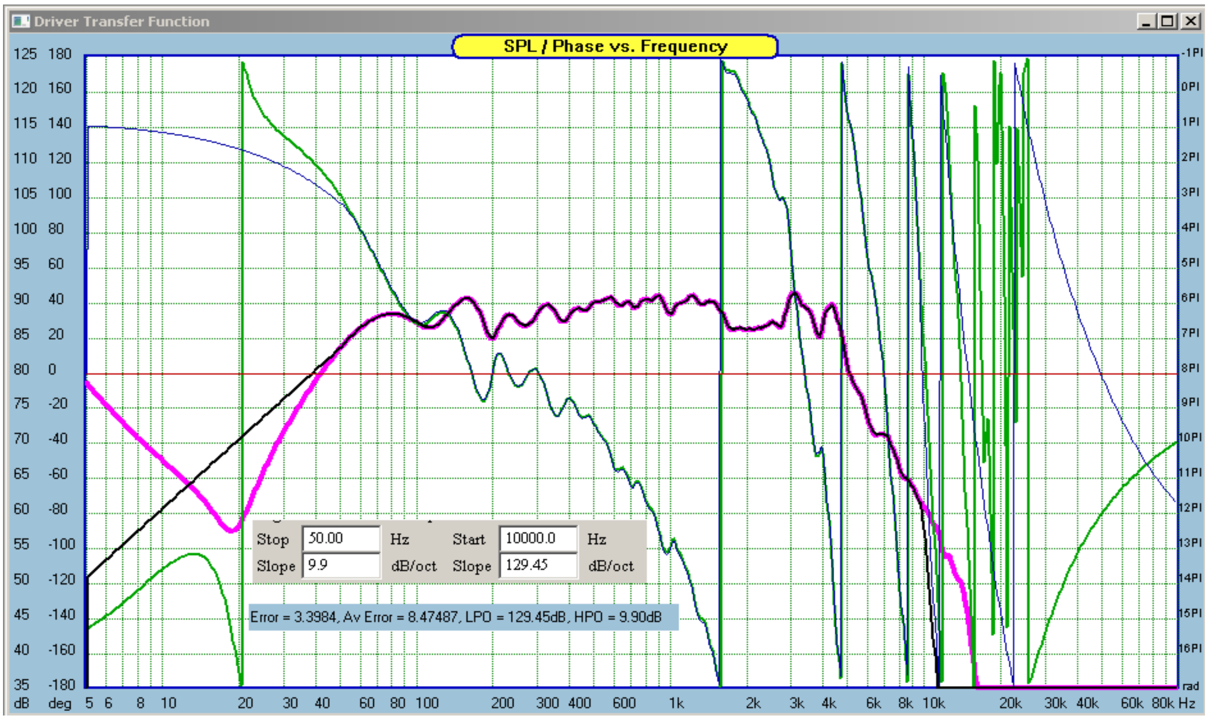


Figure 5. Example of visually very good phase match at 10kHz attachment point.

Compare the observations from above, to the “Constrained IHBT” optimization results tabulated below. The Normalized Error values increase markedly with the changes in the position of the attachment points. The Error at 10000Hz if almost **40 times higher** than at 5000Hz.

| IHBT LP Start | IHBT HP Start | Bin | Delay | LP Order | HP Order | After HBT Error | Bins | IHBT Error | Normalized Error |
|---|---|---|---|---|---|---|---|---|---|
| 4000 | 100 | 126 | -0.01 | 21.05 | 9.9 | 781.05 | 332 | 9.83 | 2.35256 |
| 5000 | 100 | 123 | 0.008 | 38.42 | 9.92 | 385.908 | 349 | 9.48 | 1.10575 |
| 6000 | 100 | 124 | 0 | 36.08 | 9.95 | 542.306 | 362 | 11.64 | 1.49808 |
| 7000 | 100 | 124 | 0.01 | 36.32 | 9.99 | 861.57 | 374 | 16.55 | 2.30366 |
| 8000 | 100 | 124 | -0.002 | 45.13 | 10.02 | 1598.38 | 384 | 18.78 | 4.16245 |
| 9000 | 100 | 123 | 0.008 | 55.55 | 10.06 | 5150.94 | 393 | 37.4 | 13.1067 |
| 10000 | 100 | 123 | -0.004 | 68.23 | 10.12 | 16123.02 | 401 | 114.57 | 40.207 |

Figure 5, showing the phase match at 10kHz attachment point, is a good example why we need numerical indication of the Cumulative Error, preferably normalized to the number of data bins used. Visually, there is nothing wrong with this phase match, and even numerically, the Normalized Error at 10kHz is 1.92, while the Averaged Error for all attachment points is 1.58.

The results tabulated above for the constrained IHBT Method show rapid increase in Error for higher attachment points.

Finally, the results for the preferred 5000kHz attachment point are the same for both methods. This is to be expected, as both methods work on same principles. Automation just makes things quicker and more accurate. Please see Figure 6 and Figure 7.

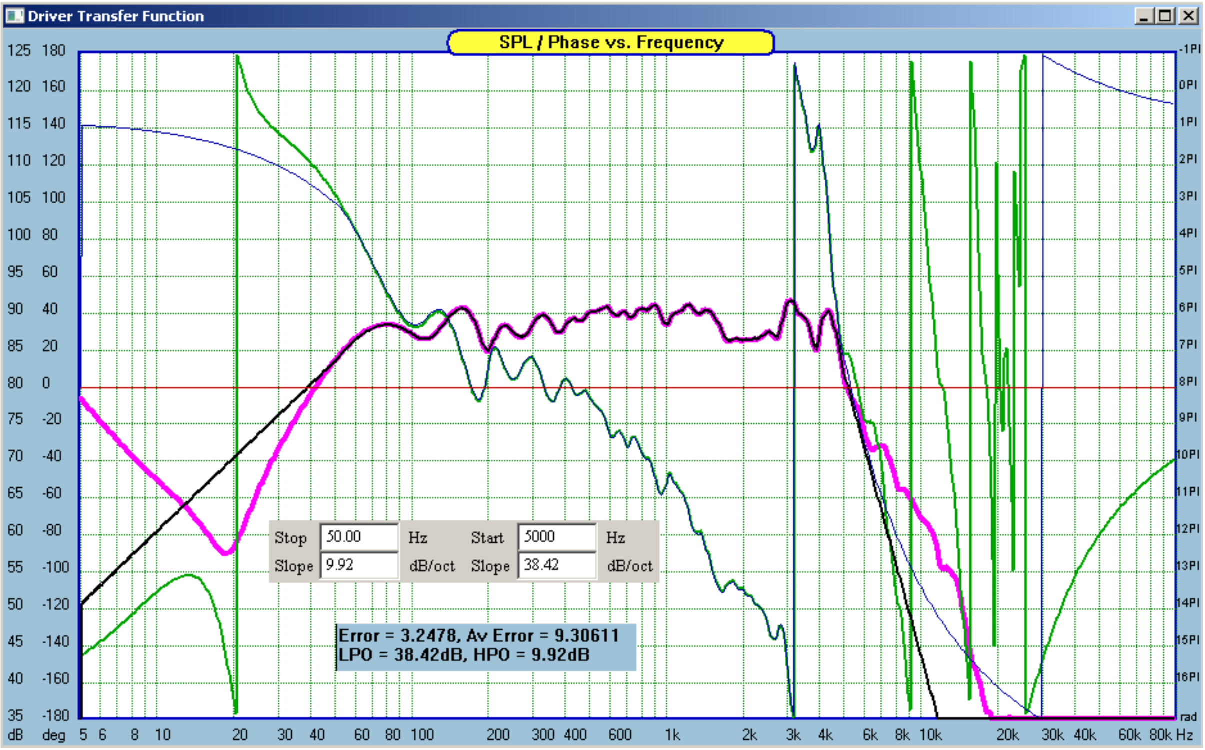



Figure 6. Optimum attachment point at 5000kHz

For 5kHz attachment point, the results from constrained IHBT and unconstrained HBT are identical. The 5kHz attachment point is quite safe to select.

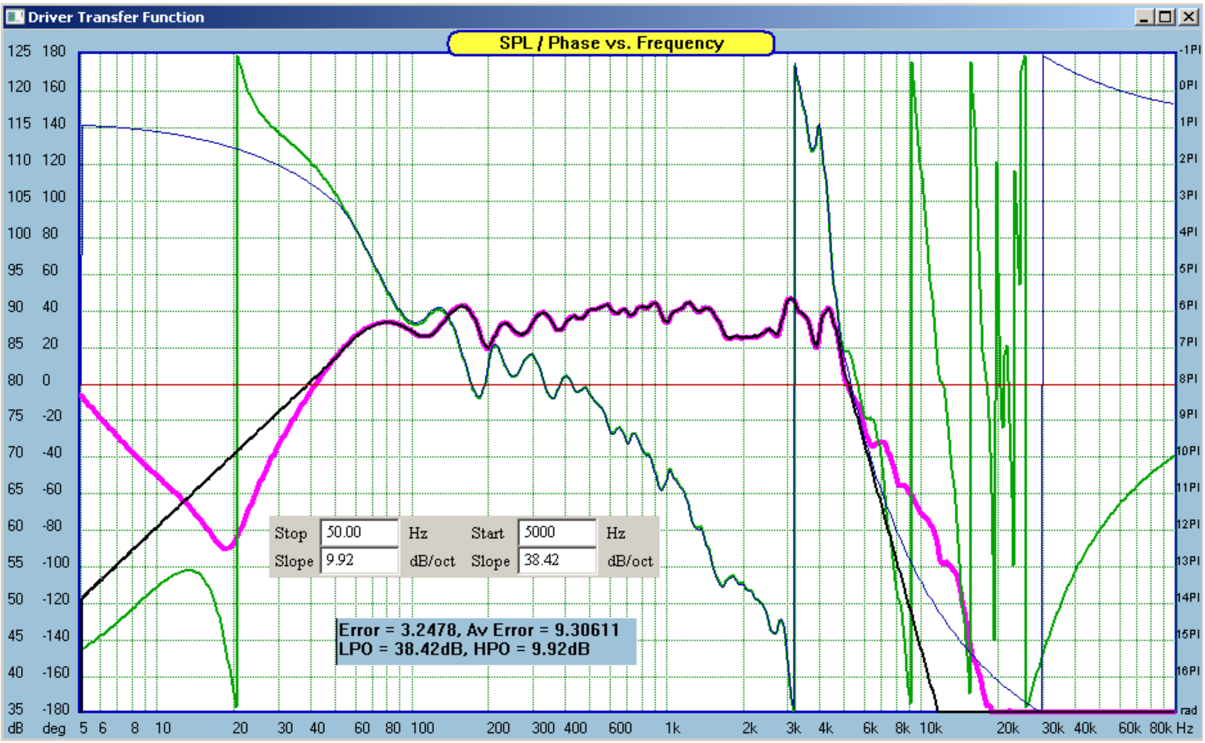



Figure 7. Optimum attachment point at 5000kHz

With the next tweeter driver, the situation is similar to the first tweeter. We observe, that frequency range 17-20kHz will meaningfully contribute to the accuracy of the process, and 22kHz frequency is too close to the limit of reliable data. Therefore, 21kHz attachment point is selected again.

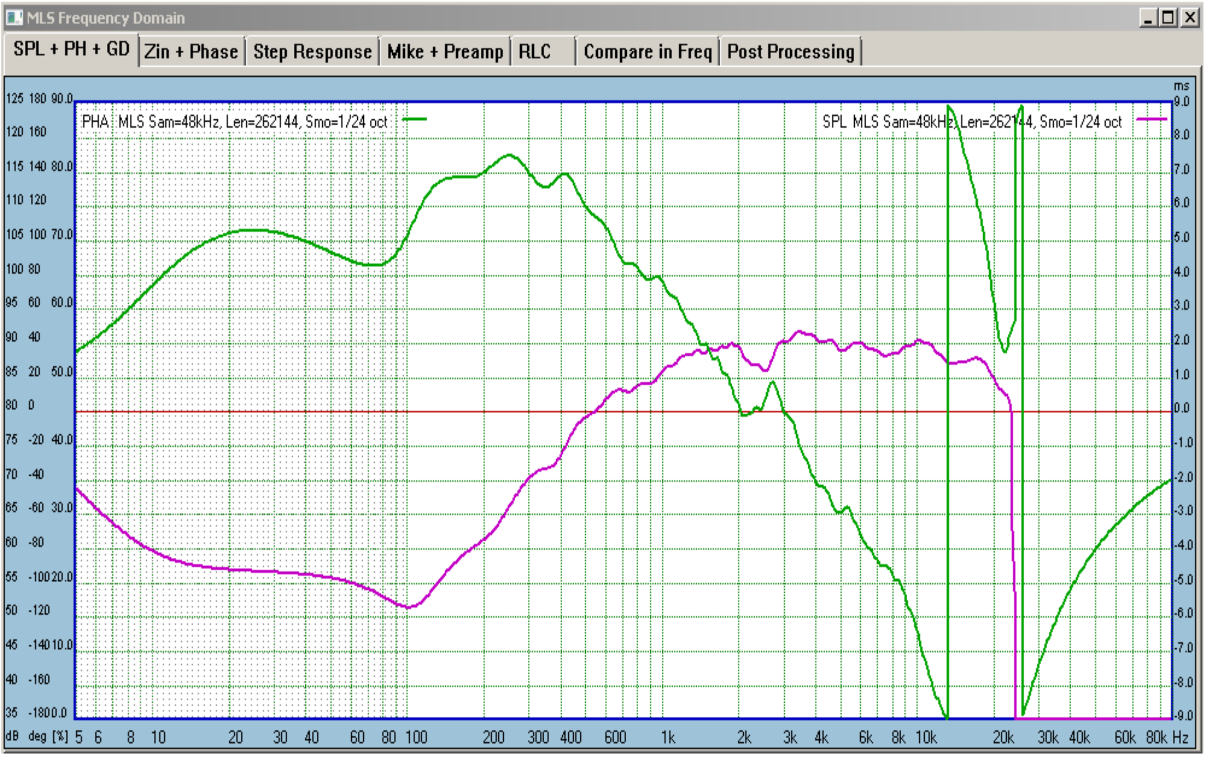



Figure 8. Example of SPL/Phase response of a tweeter driver.

Unconstrained optimization using HBT method yelds the attachment-dependant responses tabulated below

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Driver's Name | | Front_Left_Speaker | | Tweeter_50cm.res | | |
| | | | | | | | | |
| Start Bin = | 119 | Smooth = 1/24dB/oct | | | Window = 10ms | | | |
| Start HBT | 400Hz | 12dB | 22000Hz | 40dB | | | | |
| | | | | | | | | Normalized |
| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | Error |
| 17000 | 400 | 122 | -0.002 | 15.65 | 10.99 | 298.285 | 284 | 1.0503 |
| 18000 | 400 | 122 | -0.004 | 18.75 | 11.01 | 358.422 | 289 | 1.24021 |
| 19000 | 400 | 122 | -0.004 | 19.3 | 11.01 | 420.824 | 293 | 1.43626 |
| 20000 | 400 | 122 | -0.004 | 19.34 | 11.01 | 433.59 | 297 | 1.4599 |
| 21000 | 400 | 122 | -0.006 | 21.5 | 10.99 | 494.438 | 300 | 1.64813 |
| 22000 | 400 | 121 | 0.01 | 27.77 | 10.81 | 751.727 | 304 | 2.47279 |
| 23000 | 400 | 121 | 0.006 | 33.57 | 10.78 | 832.637 | 307 | 2.71217 |

Please note, that the SPL response starts to show dropping tendency towards higher frequency. This is valuable information for all employed methods, and results in more accurate phase response determination.

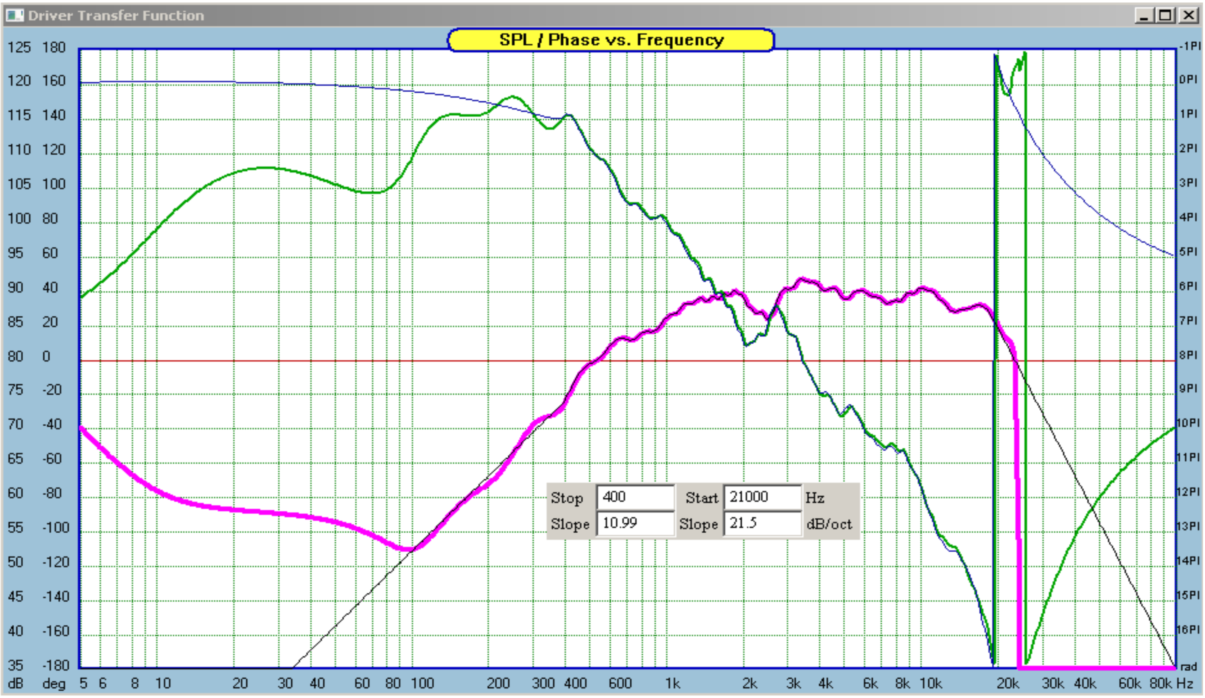


Figure 9. Phase response from unconstrained HBT method.

Constrained optimizations are tabulated below. Please note, that 22kHz Error is lower than 21kHz Error (last column). It was therefore decided to use 22kHz attachment point.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 17000 | 400 | 122 | -0.01 | 22.94 | 10.93 | 479.858 | 284 | 8.86 | 1.68964 |
| 18000 | 400 | 122 | -0.008 | 22.64 | 10.97 | 393.292 | 289 | 10.28 | 1.36087 |
| 19000 | 400 | 122 | -0.008 | 23.39 | 10.97 | 488.926 | 293 | 11.13 | 1.66869 |
| 20000 | 400 | 121 | 0.01 | 26.66 | 10.77 | 736.668 | 297 | 14.23 | 2.48036 |
| 21000 | 400 | 121 | 0.008 | 29.09 | 10.76 | 840.937 | 300 | 17.97 | 2.80312 |
| 22000 | 400 | 121 | 0.008 | 30.14 | 10.78 | 759.795 | 304 | 18.25 | 2.49933 |
| 23000 | 400 | 122 | -0.008 | 25.03 | 11.02 | 882.543 | 307 | 24.56 | 2.87473 |

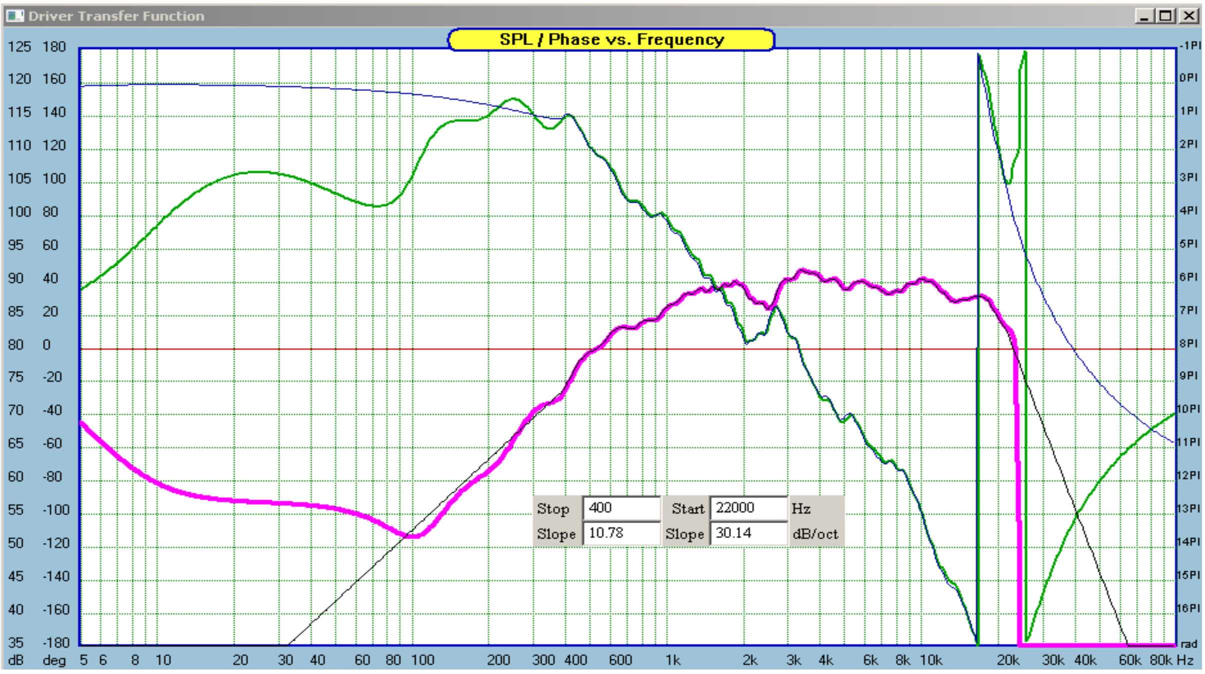


Figure 10. Phase response from constrained IHBT method

Lastly, the second woofer frequency response is shown below. Examining the measured SPL, it would be prudent to assume, that break-up region would start above 6kHz. Therefore the highest attachment point would be 6kHz.

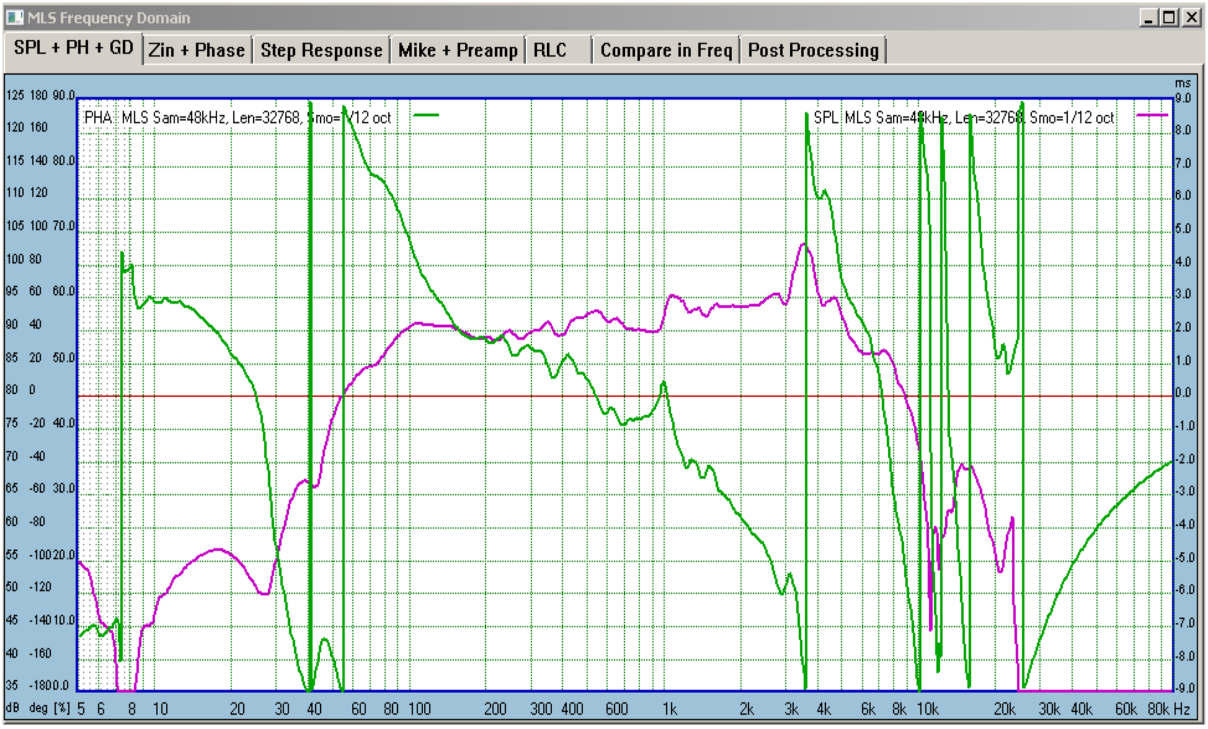



Figure 11. Example of SPL/Phase response of a tweeter driver.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Driver's Name | | IMP_RES_2.res | | | | |
| Start Bin = | 74 | Smooth = | 1/12 dB/oct | | Window = | 120ms | | |
| Start HBT | 50Hz | 18.5dB | 5000 | 40dB | | | | |
| | | | | | | After HBT | | Normalized |
| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | Error |
| 5000 | 50 | 78 | -0.012 | 24.87 | 15.74 | 2836.715 | 349 | 8.12812 |
| 6000 | 50 | 79 | 0 | 13.67 | 15.76 | 2939.151 | 362 | 8.1192 |
| 7000 | 50 | 79 | -0.002 | 16.47 | 15.76 | 3004.126 | 374 | 8.03242 |
| 8000 | 50 | 78 | -0.01 | 31.85 | 15.76 | 2947.261 | 384 | 7.67516 |
| 9000 | 50 | 77 | 0.004 | 35.21 | 15.73 | 3089.631 | 393 | 7.86166 |
| 9500 | 50 | 77 | -0.002 | 38.34 | 15.72 | 3174.942 | 397 | 7.99734 |
| 10000 | 50 | 76 | 0.008 | 44.24 | 15.7 | 3443.358 | 401 | 8.58693 |

Attachment-dependant tabulated results show the Normalized Error is very close for all attachment points. It would be very challenging indeed to discriminate between those by visually inspecting phase matches.

The 6kHz attachment point was selected for unconstrained HBT method.

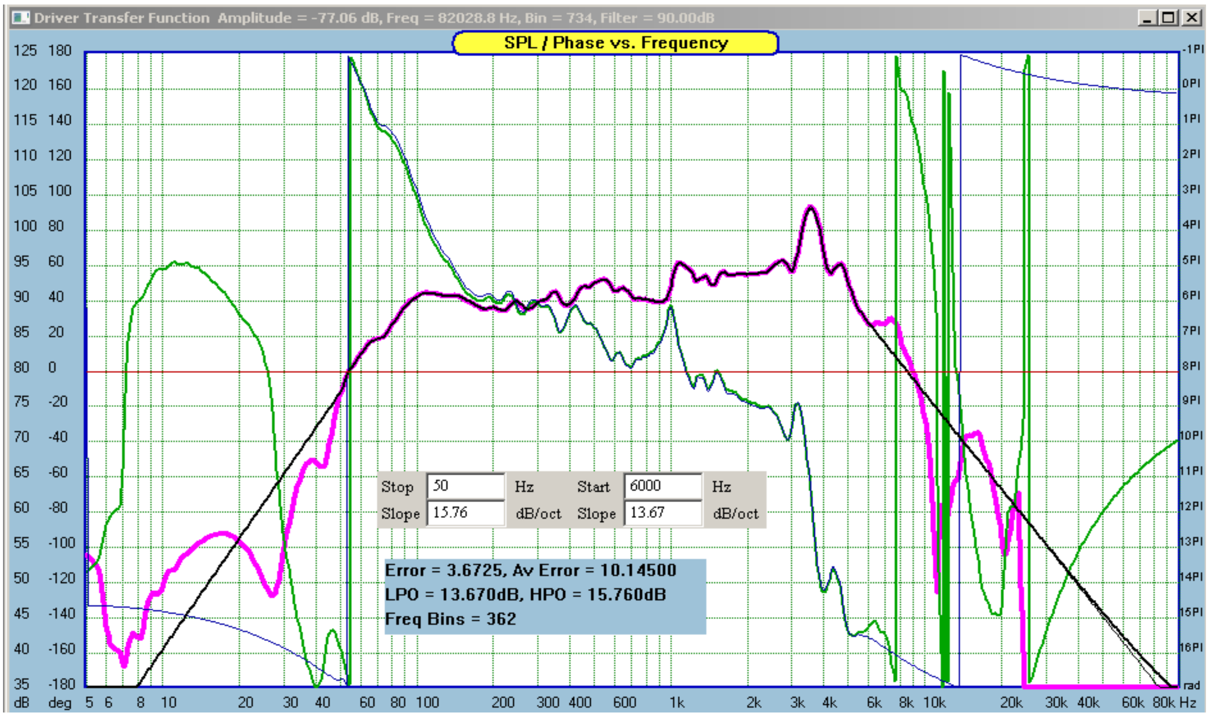



Figure 12. Phase response from unconstrained HBT method

The constrained IHBT method resulted in different slopes.

| IHBT LP Start | IHBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | IHBT Error | Normalized Error |
|---|---|---|---|---|---|---|---|---|---|
| 5000 | 100 | 76 | -0.004 | 34.02 | 15.65 | 3335.05 | 349 | 9.44 | 9.55602 |
| 6000 | 100 | 77 | 0 | 27.18 | 15.67 | 3789.58 | 362 | 11.66 | 10.4685 |
| 7000 | 100 | 78 | -0.01 | 27.39 | 15.72 | 3536.02 | 374 | 12.51 | 9.4546 |
| 8000 | 100 | 77 | -0.002 | 37.36 | 15.72 | 3171.76 | 384 | 12.99 | 8.25979 |
| 9000 | 100 | 77 | -0.006 | 40.04 | 15.71 | 3196.92 | 393 | 14.31 | 8.13466 |
| 9500 | 100 | 77 | -0.01 | 42.4 | 15.71 | 3243.13 | 397 | 15.85 | 8.16909 |
| 10000 | 100 | 76 | 0.002 | 47.44 | 15.68 | 3501.49 | 401 | 21.15 | 8.7319 |

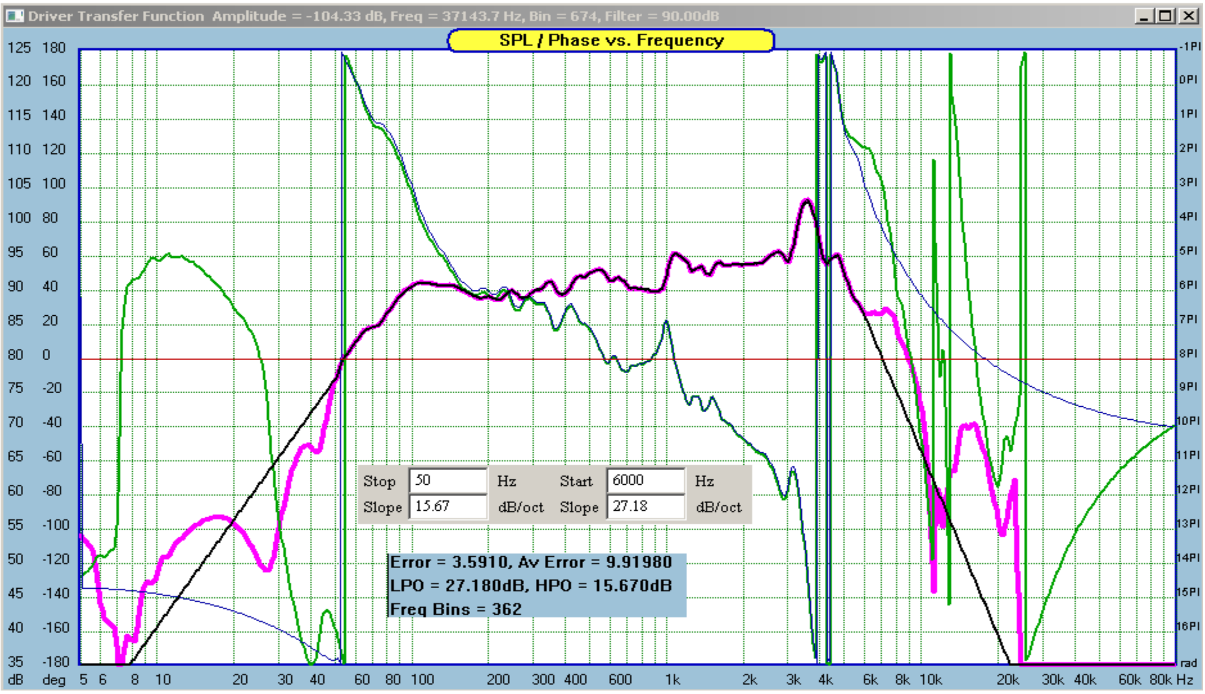



Figure 13. Phase response from constrained IHBT method

**What is the Constraint?**

As observed on several Figures presented in Part I and in this paper, manual and automated methods occasionally drift into the “difficult-to-justify” set of parameters, that also produce unusual and unexpectd plots, like Figure 2, Figure 5 and Figure 12.

The algorithms are designed to go and find the minimum error value, and they do it very efficiently, with high degree of accuracy. However, the selection of starting parameters, and the interpretation of the results lies with the human operator. For instance, starting FFT Bin for the optimization process is selected by the operator. Attachment points are selected by the operator, and as we are discussing, the constraint(s) are selected by the operator. There are other starting parameters, for sake of clarity, they will not be discussed here. Some of the reasoning behind selection of the attachment points was presented in this paper and should serve as a guidance for eliminating potential duds.

In Part I, it was suggested, that the constraint should be based on SPL response rather than phase response. Frankly speaking, I would not know how to define constraint based on phase response. SPL is much easier to deal with. We all know when the SPL curve does silly things.

Also, if we accept the SPL-based constraint, it <u>must not</u> be a “hard-limit” type of constraint. So, for instance, if one selects 40dB as a constraint for low-pass slope, it does not mean, that low-pass slope will be fixed at -40dB and the rest of the optimization must dance around this limit.

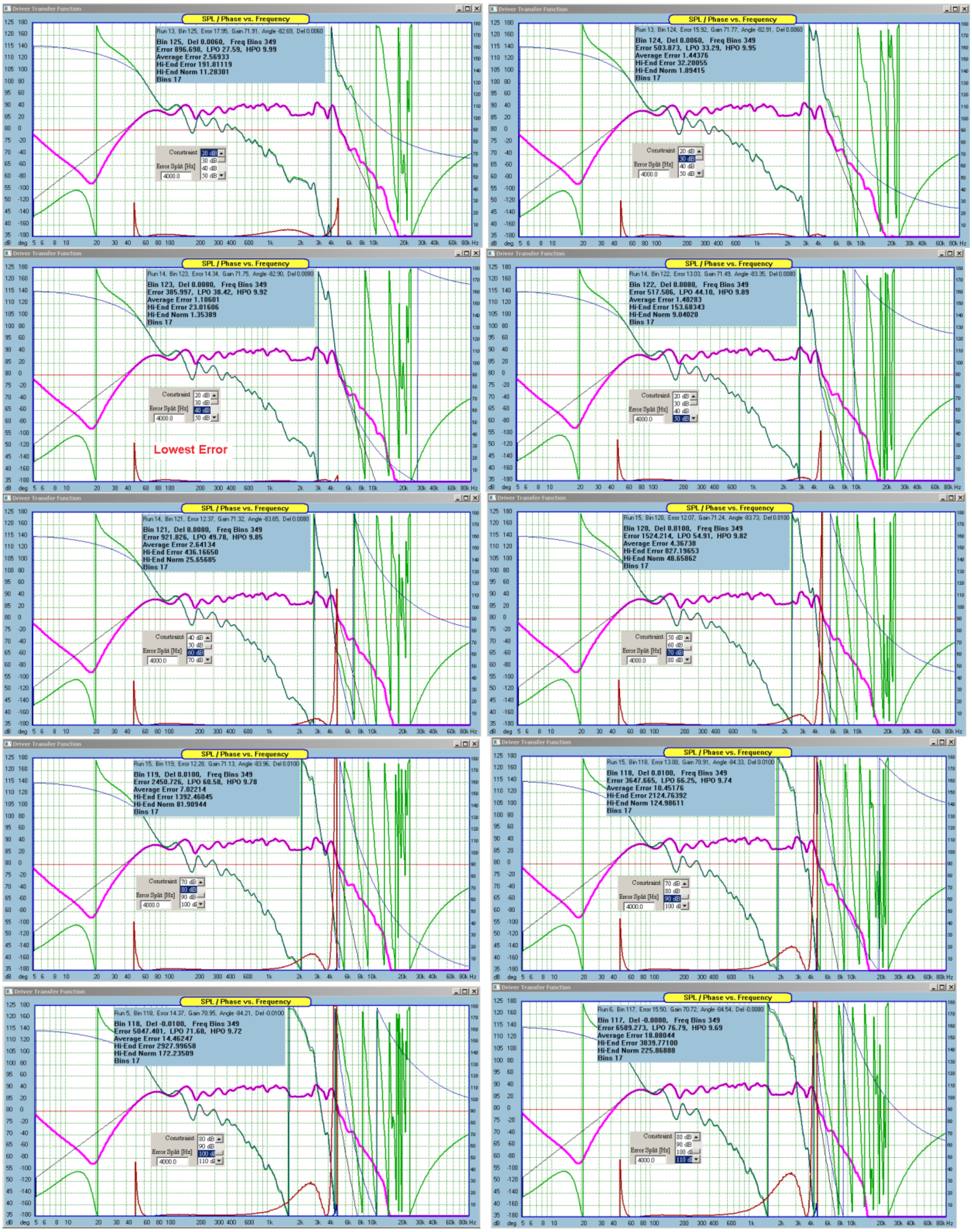
On Figure 2, as an example, the unconstrained optimization resulted in phase response corresponding to -8.97dB/oct asymptotic slope. In constrained optimization, the algorithm was asked to show the best phase matches around -30dB/oct asymptotic slope – and the algorithm presented Figure 3, with -23.43dB/oct low-pass asymptotic slope.

The unconstraint optimization results shown on Figure 12, resulted in the woofer roll-off with berely 2-nd order slope of -13.67dB/oct. While constrained optimization gave more realistic -27.18dB/oct.

Validity of the constraint can be evaluated by selecting different constraint values and running the constrained algorithm for a case presented on Figure 7.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Driver's Name | | Rear_Speaker_50cm.res | | | | | |
| IHBT Method | | | | | | | | | |
| Start Bin = | 117 | Smooth = | 1/12 dB/oct | | Window = | 25ms | | | |
| Start IHBT | 80Hz | Stop IHBT | 5000 Hz | | HBT | 50-5000Hz | | | |
| | | | | | | After HBT | | IHBT | Normalized |
| Constraint [dB] | IHBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | Error | Error |
| 20 | 80 | 125 | 0.006 | 27.59 | 9.99 | 896.698 | 349 | 17.95 | 2.56934 |
| 30 | 80 | 124 | 0.006 | 33.29 | 9.95 | 503.873 | 349 | 15.92 | 1.44376 |
| 40 | 80 | 123 | 0.008 | 38.42 | 9.92 | 385.997 | 349 | 14.34 | 1.10601 |
| 50 | 80 | 122 | 0.008 | 44.1 | 9.89 | 517.506 | 349 | 13.03 | 1.48283 |
| 60 | 80 | 121 | 0.008 | 49.78 | 9.85 | 921.826 | 349 | 12.37 | 2.64134 |
| 70 | 80 | 120 | 0.01 | 54.91 | 9.82 | 1524.214 | 349 | 12.07 | 4.36738 |
| 80 | 80 | 119 | 0.01 | 60.58 | 9.78 | 2450.726 | 349 | 12.28 | 7.02214 |
| 90 | 80 | 118 | 0.01 | 66.25 | 9.74 | 3647.665 | 349 | 13 | 10.4518 |
| 100 | 80 | 118 | -0.01 | 71.68 | 9.72 | 5047.401 | 349 | 14.37 | 14.4625 |
| 110 | 80 | 117 | -0.008 | 76.79 | 9.69 | 6589.273 | 349 | 15.5 | 18.8804 |
| 120 | 80 | 116 | -0.006 | 81.9 | 9.65 | 8346.337 | 349 | 17.6 | 23.915 |

The low-pass attachment point was selected as 5000Hz, therefore constrained optimization was run within 80Hz-5000Hz for different Constraint values 20dB – 120dB and the results are tabulated above. The Normalized Error shown on the last column, is clearly the lowest for Constraint = 40dB (green font above). The Error Curve (brown curve) progress is shown on Figure 14  below.



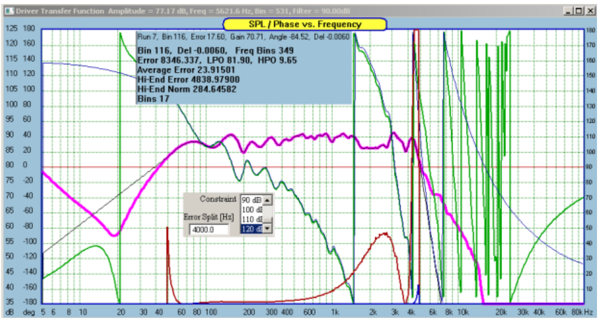



Figure 14. Excess phase calculated for different constraint values.

## Analyzing Errors

Having the Cumulative Error value and particularly the Normalized Cumulative Error Value assisting in phase extraction decisions is of a great help. But there is more to it.

In the next set of tabulated results, the Cumulative Error was split into Hi-End Error (above 4000Hz) and Low-End Error (below 4000Hz).

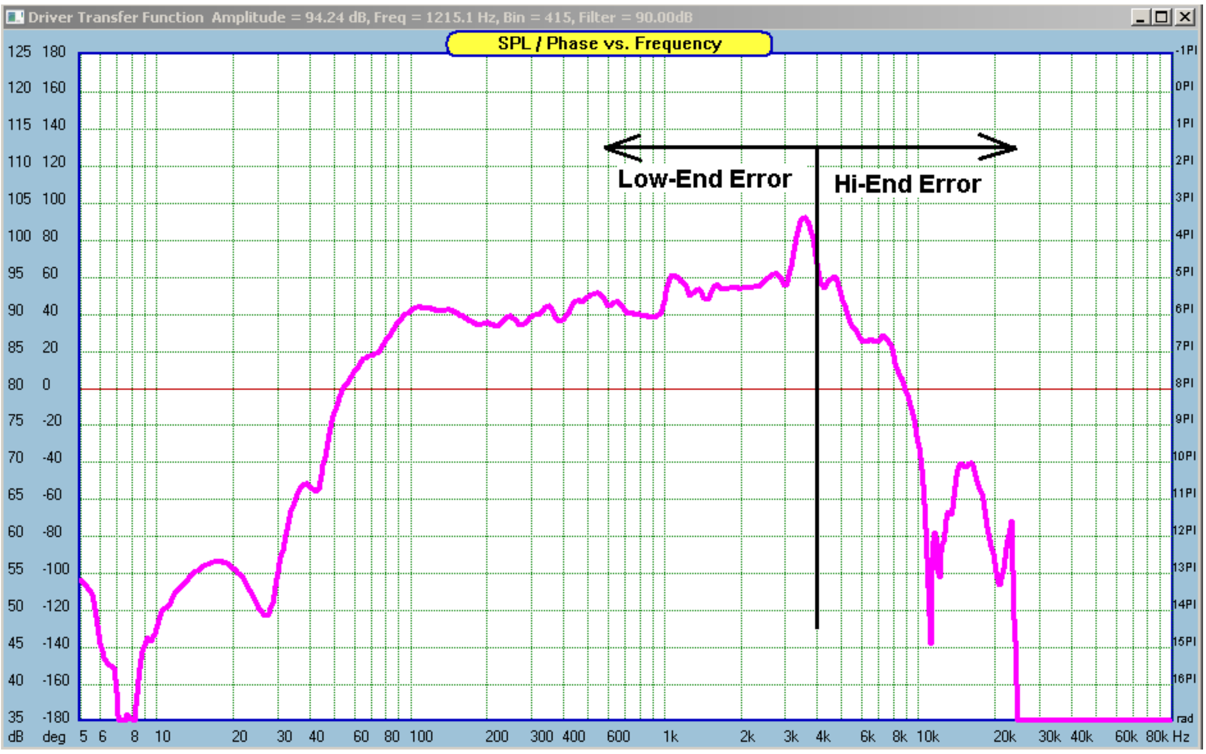



Figure 15. Example of splitting Error into two regions.

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | 4kHz = 505 bin | | | |
| Start Bin = | 74 | Smooth = | 1/12 dB/oct | | Window = | 120ms | | | | | | | | | |
| Start HBT | 50Hz | 18.5dB | 5000 | 40dB | | | | | | | | Hi-End | | | |
| | | | | | | After HBT | | Normalized | | Hi-End | Hi-End | Normalized | | Low-End | Normalized |
| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HP Order | Error | Bins | Error | | Error | Bins | Error | | Error | Low-End Error |
| 5000 | 50 | 78 | -0.012 | 24.87 | 15.74 | 2836.715 | 349 | 8.12812 | | 5.48422 | 17 | 0.3226 | | 2831.2308 | 8.527804 |
| 6000 | 50 | 79 | 0 | 13.67 | 15.76 | 2939.151 | 362 | 8.1192 | | 57.2272 | 30 | 1.90757 | | 2881.9239 | 8.680494 |
| 7000 | 50 | 79 | -0.002 | 16.47 | 15.76 | 3004.126 | 374 | 8.03242 | | 86.8848 | 42 | 2.06868 | | 2917.2412 | 8.786871 |
| 8000 | 50 | 78 | -0.01 | 31.85 | 15.76 | 2947.261 | 384 | 7.67516 | | 59.4415 | 52 | 1.14311 | | 2887.8195 | 8.698251 |
| 9000 | 50 | 77 | 0.004 | 35.21 | 15.73 | 3089.631 | 393 | 7.86166 | | 128.042 | 61 | 2.09906 | | 2961.5886 | 8.920448 |
| 9500 | 50 | 77 | -0.002 | 38.34 | 15.72 | 3174.942 | 397 | 7.99734 | | 216.979 | 65 | 3.33814 | | 2957.9632 | 8.909528 |
| 10000 | 50 | 76 | 0.008 | 44.24 | 15.7 | 3443.358 | 401 | 8.58693 | | 408.886 | 69 | 5.92589 | | 3034.4716 | 9.139975 |
| | | | | | | | | | | | Growth | 18.3692 | | Growth | 1.071785 |

Probably the most interesting aspect of this analysis is the growth of Hi-End Error and Low-End Error calculated for different low-pass attachment points.

The growth of Normalized Low-End Error is only **1.0717 times** for attachments 5kHz-10kHz
The growth of Normalized High-End Error is a wooping **18.36 times** for attachments 5kHz-10kHz. It is important to calculate Normalized values, because they account for increased number of frequency bins at high-end of the bandwidth.

It is evident, that almost all the growth in Error values comes from drifting into the break-up region. On the next Figure 16, the plots also incorporate the Error Value ( thick brown line) versus frequency plots. The attachemnt points start at 5kHz and end at 10kHz.

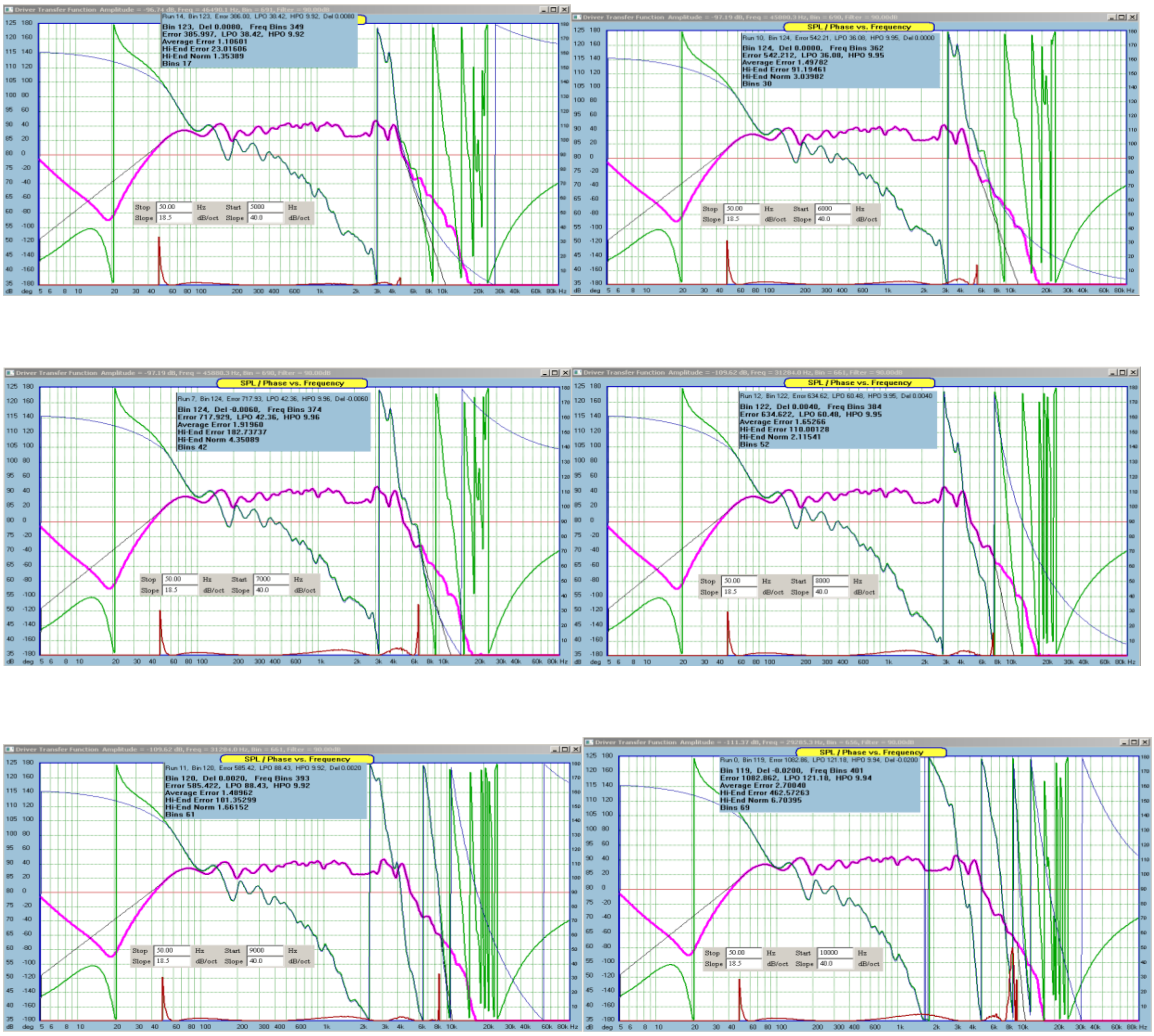
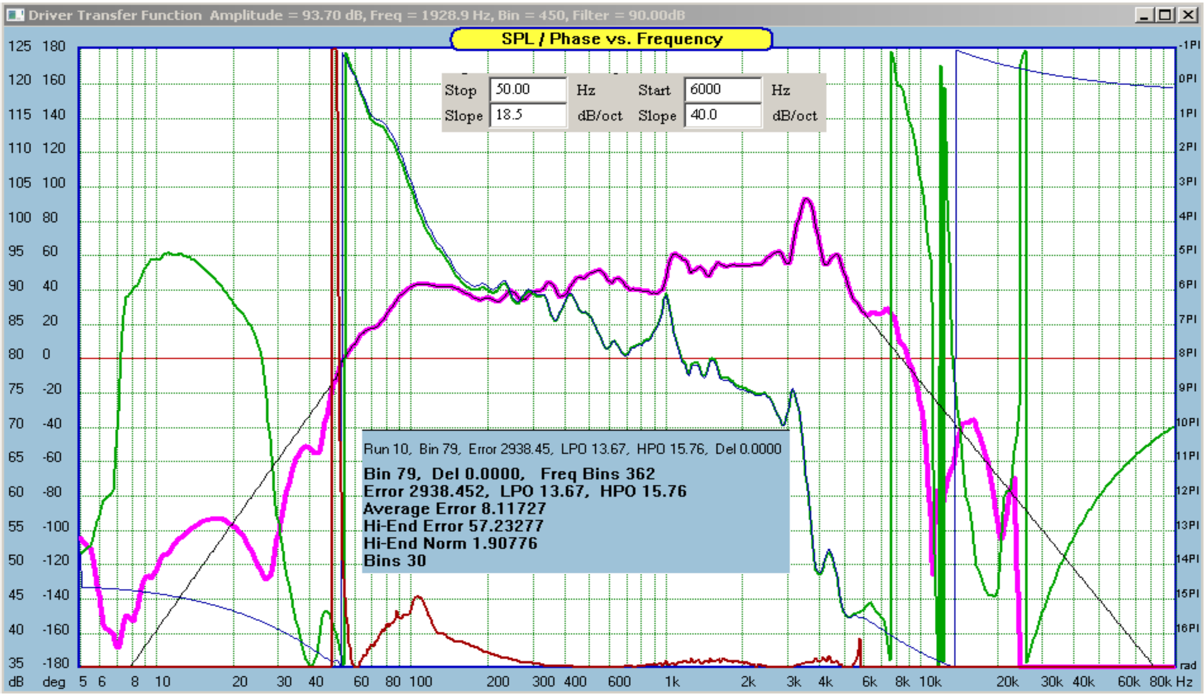


Figure 16. Error split into two regions.

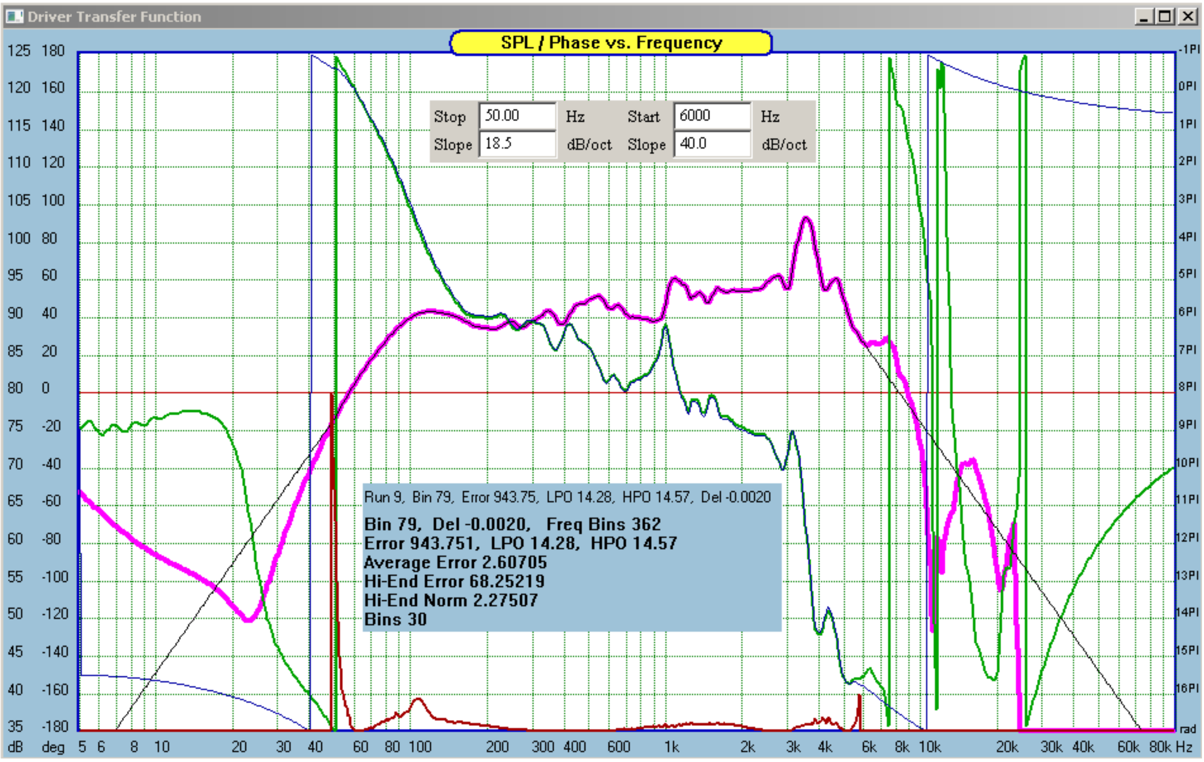
There is very little change in Error Value the lower frequency range. All the growth comes from moving the attachment point into the break-up region.

The Error Value in the low-frequency region will be affected by Windowing and Smoothing parameters. In this case, the effect is the opposite, and the Error does not change very much at the high end, but (as in the example below), the Error is 3 times lower at low end of the frequency range for narrower FFT Window of 30ms.

FFT Window = 120ms, Smoothing = 1/12dB/oct




FFT Window = 30ms, Smoothing = 1/12dB/oct




Figore 17. Low-frequency Error depends on Windowing and Smoothing

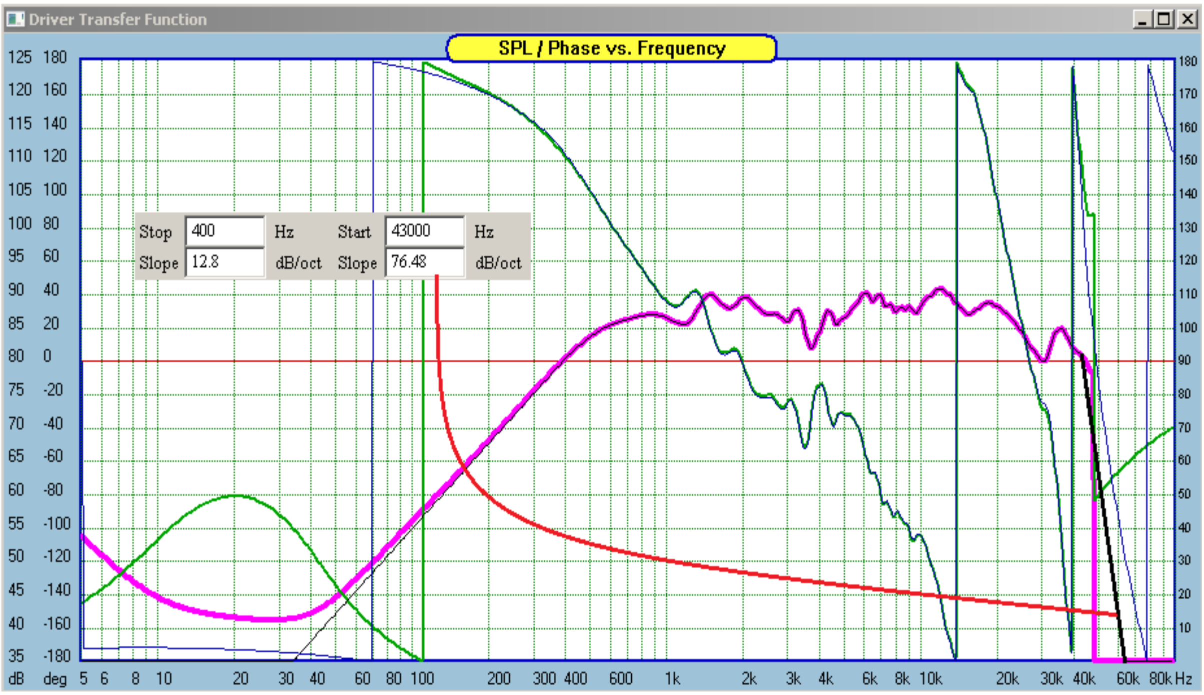
Finally, we examine one more tweeter driver. This one has been sampled at 96kHz, using tabulated results for different attachment point, shown below.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Driver's Name | | Raw_Tweeter_0.0deg_100cm.res | | | | | |
| | | | | | | | | | |
| Start Bin | 323 | Smooth= | 1/48dB/oct | | Window = | 5ms | Time Step = | | 0.001ms |
| Start HBT | 400 | 12 | 40000 | 40dB | | | | | |
| | | | | | | | | Normalized | |
| HBT LP Start | HBT HP Start | Bin | Delay | LP Order | HO Order | Error | Bins | Error | |
| 38000 | 400 | 327 | -0.002 | 50.93 | 12.84 | 500.299 | 345 | 1.45014 | |
| 39000 | 400 | 327 | -0.003 | 53.84 | 12.83 | 535.64 | 347 | 1.54363 | |
| 40000 | 400 | 327 | -0.004 | 56.95 | 12.82 | 539.388 | 349 | 1.54552 | |
| 41000 | 400 | 326 | 0.005 | 61.47 | 12.82 | 507.413 | 351 | 1.44562 | |
| 42000 | 400 | 326 | 0.005 | 67.67 | 12.81 | 455.586 | 353 | 1.29061 | |
| 43000 | 400 | 326 | 0 | 76.48 | 12.8 | 418.701 | 355 | 1.17944 | |
| 44000 | 400 | 326 | -0.001 | 79.88 | 12.8 | 432.41 | 356 | 1.21463 | |
| 45000 | 400 | 326 | -0.004 | 89.37 | 12.8 | 559.564 | 358 | 1.56303 | |
| 46000 | 400 | 326 | -0.004 | 91.45 | 12.8 | 1076.5 | 360 | 2.99028 | |
| 47000 | 400 | 326 | -0.004 | 92.44 | 12.83 | 1603.311 | 361 | 4.4413 | |

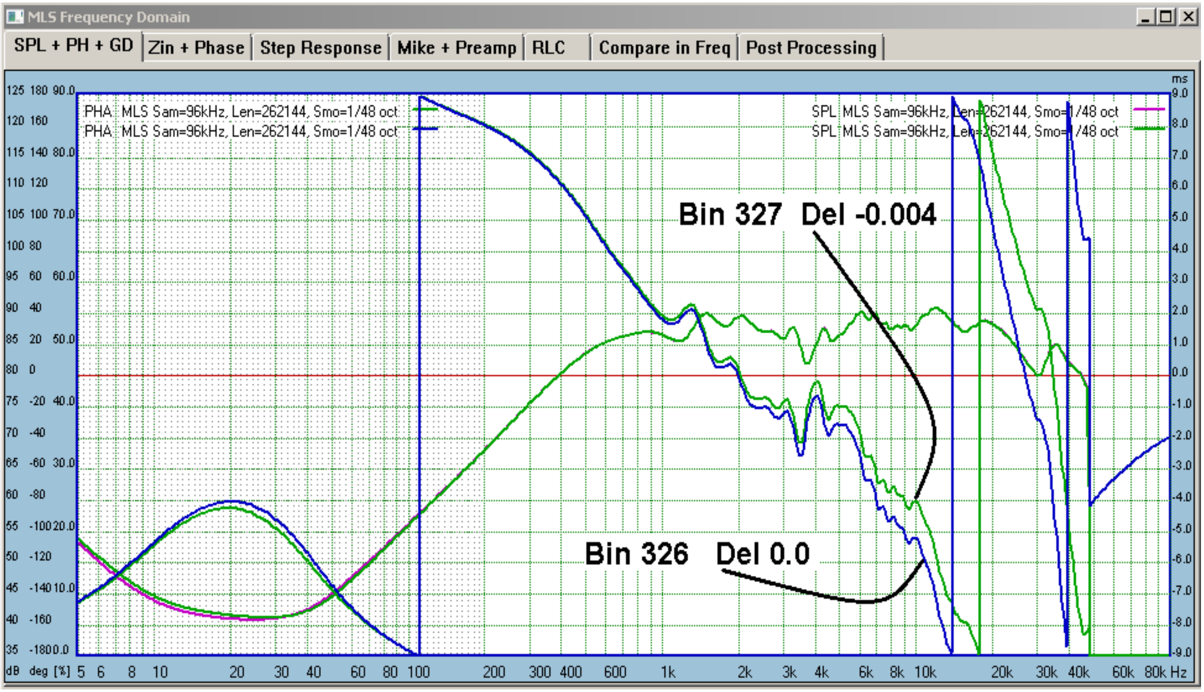
The SPL and phase response corresponding to the minimum Normalized Error of 1.17944. The final low-pass SPL slope is presented as black thick line of -76dB/oct roll-off.

It seems to be quite fast roll-off, and the combined SPL response looks unnatural. It would be very challenging to visually discriminate for the best phase match between 38kHz right up to 45kHz attachment points. The errors are very close to each other indeed.

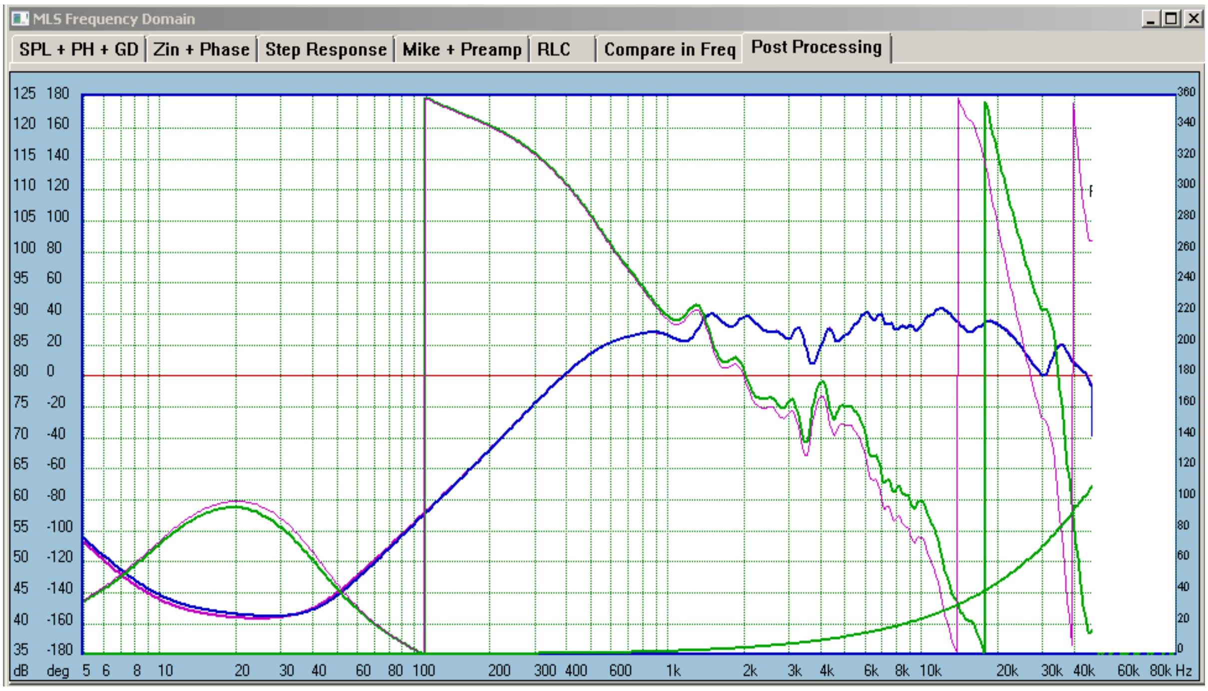
Even steeper roll-offs are require to satisfy minimum errors at 44kHz – 47kHz. At 47kHz attachment point the roll-off is -92.44dB/oct.



The phase responses for 40kHz and 43kHz attachment points using Un-Constrained HBT method are plotted below.




Phase responses for 40kHz and 43kHz attachment




It is observable, that the phase difference is 48 deg at 20kHz (green line).

Similarly, the same tweeter was examined using Constrained IHBT Method. Two full runs were conducted. One for 40kHz attachment point and one for 43kHz attachment point

Run 1: From 400Hz To 40000Hz

```
B 327, Del 0.003, E 1434.30, Con 40, NE 4.1097, LP 41.94, HP 12.91, E-LP -1.94
B 327, Del 0.001, E 983.08, Con 45, NE 2.8168, LP 46.23, HP 12.88, E-LP -1.23
B 327, Del 0.000, E 816.17, Con 50, NE 2.3386, LP 48.38, HP 12.87, E-LP 1.62
B 327, Del -0.002, E 599.76, Con 55, NE 1.7185, LP 52.66, HP 12.85, E-LP 2.34
B 327, Del -0.003, E 550.19, Con 60, NE 1.5765, LP 54.82, HP 12.84, E-LP 5.18
B 327, Del -0.004, E 539.44, Con 65, NE 1.5457, LP 56.95, HP 12.82, E-LP 8.05
B 326, Del 0.005, E 616.70, Con 70, NE 1.7671, LP 59.94, HP 12.80, E-LP 10.06
B 326, Del 0.004, E 706.04, Con 75, NE 2.0230, LP 62.08, HP 12.78, E-LP 12.92
B 326, Del 0.002, E 1001.26, Con 80, NE 2.8689, LP 66.36, HP 12.76, E-LP 13.64
B 326, Del 0.001, E 1207.32, Con 85, NE 3.4594, LP 68.51, HP 12.76, E-LP 16.49
B 326, Del 0.000, E 1452.35, Con 90, NE 4.1614, LP 70.66, HP 12.75, E-LP 19.34
B 326, Del -0.001, E 1736.35, Con 95, NE 4.9752, LP 72.79, HP 12.74, E-LP 22.21
B 326, Del -0.003, E 2415.48, Con 100, NE 6.9211, LP 77.08, HP 12.71, E-LP 22.92
B 326, Del -0.004, E 2814.04, Con 105, NE 8.0631, LP 79.22, HP 12.71, E-LP 25.78
B 325, Del 0.005, E 3455.32, Con 110, NE 9.9006, LP 82.26, HP 12.68, E-LP 27.74
```

Run 2: From 400Hz To 43000Hz

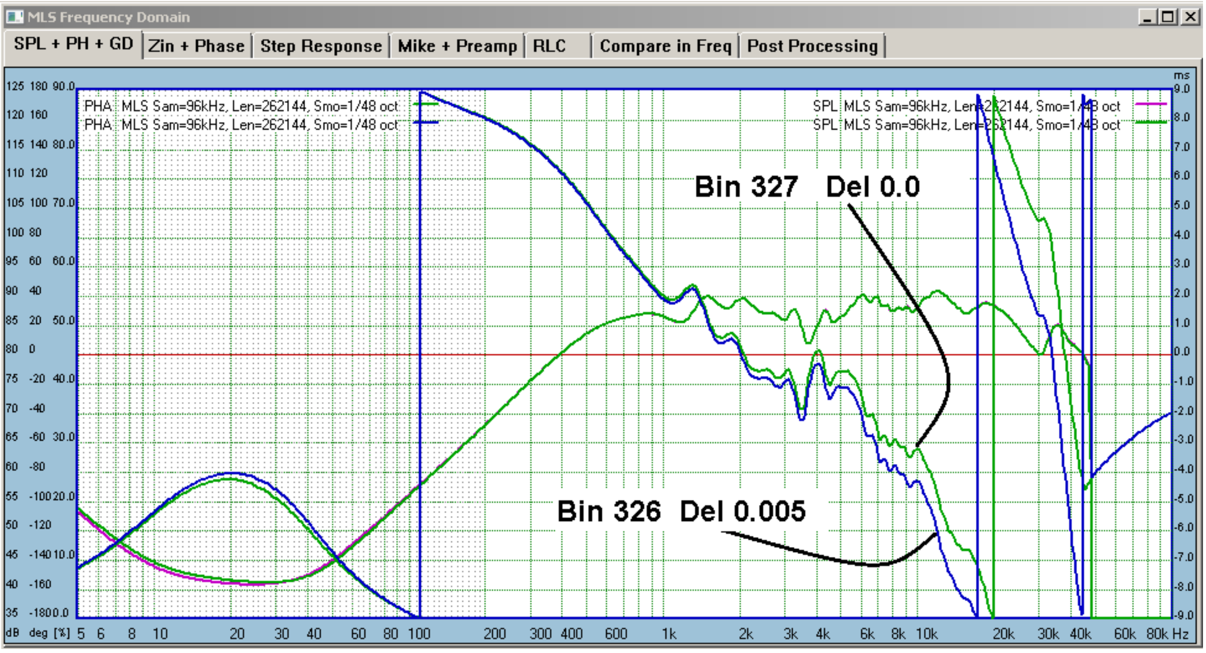
```
B 327, Del 0.000, E 2797.17, Con 40, NE 7.8794, LP 52.28, HP 12.93, E-LP -12.28
B 327, Del -0.001, E 2356.65, Con 45, NE 6.6385, LP 54.59, HP 12.92, E-LP -9.59
B 327, Del -0.002, E 1961.28, Con 50, NE 5.5247, LP 56.91, HP 12.90, E-LP -6.91
B 327, Del -0.003, E 1611.23, Con 55, NE 4.5387, LP 59.24, HP 12.90, E-LP -4.24
B 327, Del -0.004, E 1306.72, Con 60, NE 3.6809, LP 61.55, HP 12.89, E-LP -1.55
B 326, Del 0.005, E 995.68, Con 65, NE 2.8047, LP 64.89, HP 12.87, E-LP 0.11
B 326, Del 0.003, E 629.74, Con 70, NE 1.7739, LP 69.53, HP 12.85, E-LP 0.47
B 326, Del 0.002, E 513.85, Con 75, NE 1.4475, LP 71.84, HP 12.83, E-LP 3.16
B 326, Del 0.001, E 443.66, Con 80, NE 1.2497, LP 74.15, HP 12.82, E-LP 5.85
B 326, Del 0.000, E 418.70, Con 85, NE 1.1794, LP 76.48, HP 12.81, E-LP 8.52
B 326, Del -0.001, E 438.50, Con 90, NE 1.2352, LP 78.79, HP 12.79, E-LP 11.21
B 326, Del -0.003, E 612.59, Con 95, NE 1.7256, LP 83.42, HP 12.77, E-LP 11.58
B 326, Del -0.004, E 767.18, Con 100, NE 2.1611, LP 85.73, HP 12.77, E-LP 14.27
B 326, Del -0.005, E 966.67, Con 105, NE 2.7230, LP 88.05, HP 12.75, E-LP 16.95
B 325, Del 0.004, E 1314.76, Con 110, NE 3.7036, LP 91.34, HP 12.74, E-LP 18.66
```

The SPL constraint ( Con XX above) for low-pass slope was varied from 40dB to 110dB and IHBT method was run for all these values. Resulting FFT Bin values ( B XXX above ), Error Values ( E XXX.X above ), Normalized Error (NE X.XXXX above ) Low-Pass slope ( LP XX.XX above ) and the difference between Constraint – Low-Pass slope ( E-LP X.XX) were tabulated.

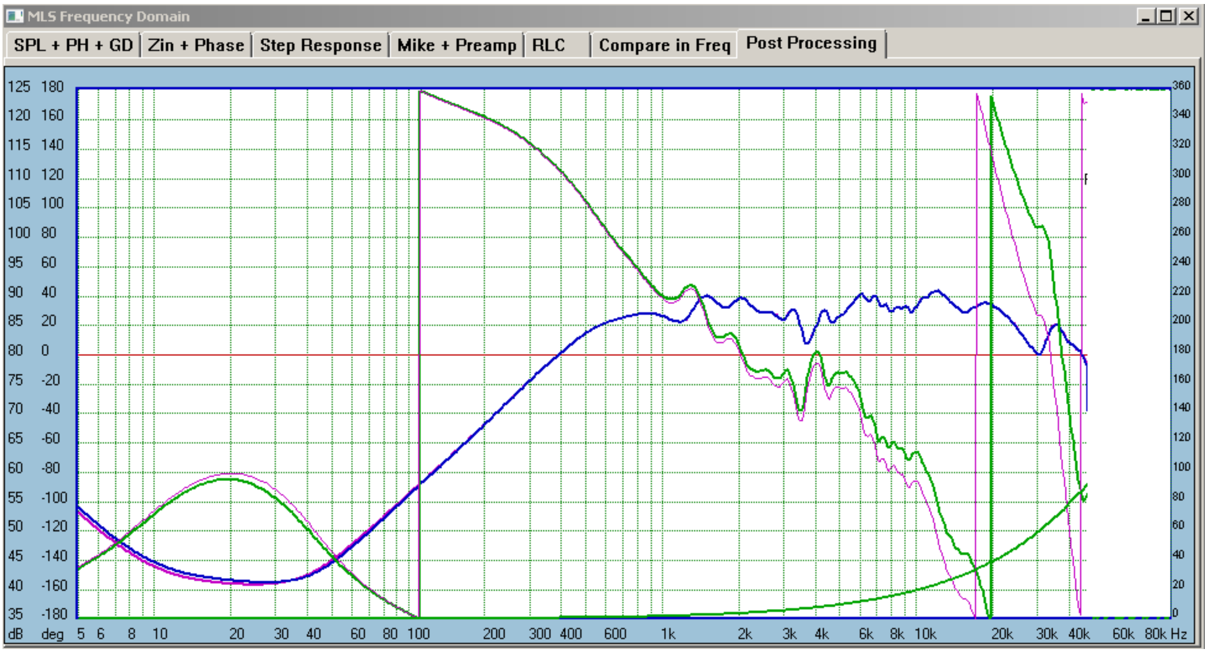
It is observable, that E-LP parameter can be negative, zero (or close to zero ) or positive. **When the E-LP = 0, the constraint is not active, and the process can be considered unconstrained**.

For E-LP = 0.11 ( close to zero), the algorithm recommends 64dB/oct Low-Pass slope.

The recommended excess phase values were plotted for both attachment points 40kHz and 43kHz, and compared – see plots below.




40kHz and 43kHz Constrained IHBT method




Constrained IHBT Phase Difference 38 deg at 20kHz (green line).

It is observable, that Constrained HBT method offered reduction in excess phase difference from 48 deg to 38deg at 20kHz, which is by 20.8%.

This is not a large value, but combined with the recommended slope of 64dB/oct (vs 76dB/oct) seems to be shifting the excess phase result in the right direction.

One final interesting observation: the averaged value of 4 slopes for attachment poins between 40kHz, 41kHz, 42 kHz and 43kHz is equal to 65.64dB/oct. It may not be a bad idea, to select 41.5kHz attachment point and 65dB low-pass slope. Both methods seem to be pointing to the same conclusion.

**Conclusions**

Once the measurement process using MLS or ESS methods is completed, you are presented with SPL and Phase responses. Next comes the all-important question: “And now what?”.

Minimum-Phase extraction process can be rather complicated process. In order to lessen the burden, a selection of tools and methods has been devised, encompasing (1) inclusion of all measurement processes and measurement options, (2) automation, (3) various numerical error presentations, (4) Error vs. frequency display curve and (5) common senese analysis of all these results. It is advisable to use as many tools as there are available to estimate the elusive minimum-phase response of a measured driver.

Most often, the smallest error will be the best indication of the phase correctness. Other times, bizzare SPL curve will eliminate some results, and what’s left would be the correct phase outcome. Finally, analysis of the measured SPL/Phase curve will offer some clues as to what the next step should be in selecting automation options, particularly when strategically selecting the attachment points. Some more insight can be provided by the IHBT method here.

When analysing the results, it would be prudent, to take into account sensitivities of the whole phase extraction to changes in parameters of measurement process. For instance, it would be incorrect to compare phase errors calculated for different windowing or smoothing parameters. The two Windowing example figures presented before show the Average Error = 2.60705 for 30ms FFT Window, as opposed to Average Error = 8.11727 for 120ms FFT Windows. Parameters of the measurement process need to be selected based on sound measurement practices, and kept constant for all phase extraction activities.

Cumulative Error is a good indication of where the things are going globally. Normalized Error helps to determine if extending the bandwidth via moving the attachment points causes the error to grow unusually large. For instance, if the phase difference between measured and HBT-drived phase is a steady 1 degree per frequency point, the Cumulative Error over 1 data point will be equal to 1. Cumulative Error for extended bandwith of 10 data points will be equal to 10. But Normalized error will be still equal to 10/10 = 1. So, it will be the Normalized Error, that will alert you to unusually large phase gaps. Examining the Error vs. frequency plot is also beneficial.

Normalized Error vs. Constraint tabulated results and Normalized Error vs Attachment Point tabulated results would ideally reconcile with the same location of the FFT window and delay, and therefore the same excess phase.

The HBT Method and IHBT Method described in Part I and in this paper above, work essentially off the same principles, therefore, they produce the same results. The methods were designed to compliment each other, and act as a consistency check for each other. Any differences could be attributed to the finite time-step resolution in algorithmic implementation and are negligible. The processes allow the user to extract minimum-phase response with +/- 1usec accuracy when used with 96kHz sampling.

Thank you for reading
Bohdan