**Chapter 15. Finite Element Analysis**

**The FEM.**

Before we proceed further, a few words of explanation on the FEM are needed. This concept has been around for several decades and lends itself particularly well to solving Helmholtz equation for complex volume shapes. Within a volume V, enclosed by a surface S, the pressure p must satisfy wave equation:

$$\nabla^2 p + (\frac{\omega^2}{c^2})p = 0 \qquad \text{c = speed of sound, } \omega \text{ =frequency of vibrations}$$

On the hard surface, S, the normal velocity $\partial p / \partial n = 0$. The solution to the above problem, expressed by an equivalent variational principle is:

$$\delta \int_V \frac{1}{2}[(\nabla p)^2 - (\omega^2 / c^2)p^2]dV = 0$$

The volume in question is divided into a large number of smaller elements, each having several nodes. We concentrate on an 8-node (8-corner) “brick” element with linear shape functions approximating pressure distribution within the element. The bricks are placed together to approximate the required volume and shape of the room. The solution of the problem is expected to assign each node a pressure value for every room mode (eigenvalue). When shape functions of the element are introduced into the variational principle, the equation is reduced to matrix eigenvalue problem:

$$[K - \frac{\omega^2}{c^2}M]\{p\} = 0$$

K is the “stiffness” matrix, M is the “inertia” or mass matrix. Now the matrix equation can be solved by standard eigenproblem methods.

Generally, the user of an FEM program is dependent on the detailed knowledge about theories, algorithms and assumptions behind the program for the proper selection of models and algorithms. The FEM knowledge base is huge and readily available, but the immediate question is: do you need to become an FEM expert if you only need to analyse your room acoustics once?. Please read on.

**Analyzing FEM output**

**Note: SoundEasy offers two FEM screens for more complete analysis of your room acoustics and enclosure driving point impedance. In order to save memory, the screens share same of the data space. We therefore recommend, that you perform analysis on one screen and SAVE results of your work to disk before opening the other FEM screen. If you start new analysis, please ALWAYS use "Clear" button on each screen to ensure consistent starting point for the FEM process.**

In general, the loudspeakers tend to activate those modal frequencies, which have partial or full maximum at this particular location. Conversely, room modes that have null at the loudspeaker location can not be energized. Because some absorption and transmission losses are always present in the room, the modes can not develop infinite pressure patterns and will always decay when the source of excitation is removed. However, as long as there are reflections in the closed volume of the room, the modes will be present. A solution that is always recommended is to install bass traps. If the economical and aesthetic factors are on your side the problem can be solved, or at least partially eradicated.

Bass traps will greatly reduce reflections, thus preventing the standing waves phenomenon taking full effect and you will gain much more freedom in positioning the speakers. You simply kill the problem right where it starts. We have browsed the Internet for some help in this area and we can testify that there are several companies offering good solutions to this problem. Room modes that need to be looked at are typically located below 100 Hz, so you need to make sure that bass traps you wish to implement are really efficient at such low frequencies. If, for whatever reasons, the bass traps are not an option for you we suggest careful review of the modal pressure plots. From this moment onward, we are not trying to solve the problem, but rather select the “lesser evil”. Good FEM simulation software usually calculates all room modes and associated pressure patterns all at once, so when this lengthy process is completed you only need to flip through the list of modal frequencies to get the pressure pattern displayed on the screen. What you looking for are the location of pressure peaks and nulls for each mode (frequency). Larger loudspeaker systems are typically made as floor standing and for purely aesthetic reasons you will not place them in the middle of the room, but rather against the wall or even in the corner. From the pressure plots, you will notice that pressure peaks also like walls and corners.

Therefore, if you end up energizing room modes, you may as well energize as many of them as possible. If the room has many modes spaced evenly, the sound field tends to be smoother than in a room with only a few well-separated modes. The latter room sounds boomy at those frequencies. Next, the pressure plot analysis should reveal the locations of pressure nulls. This is essential, as one of the issues of prime importance here is to avoid cancellation of sound. Having located the nulls, you know at least, which areas of the room to avoid for quality listening. Many of us are would rather accept some amplification of sound than cancellation of sound (and our effort). Moreover, if this particular mode is too loud you can always use electronic equalization to trim it down. The downside of living with the room modes is that the acoustic frequency response (loudspeaker + room) WILL be irregular.

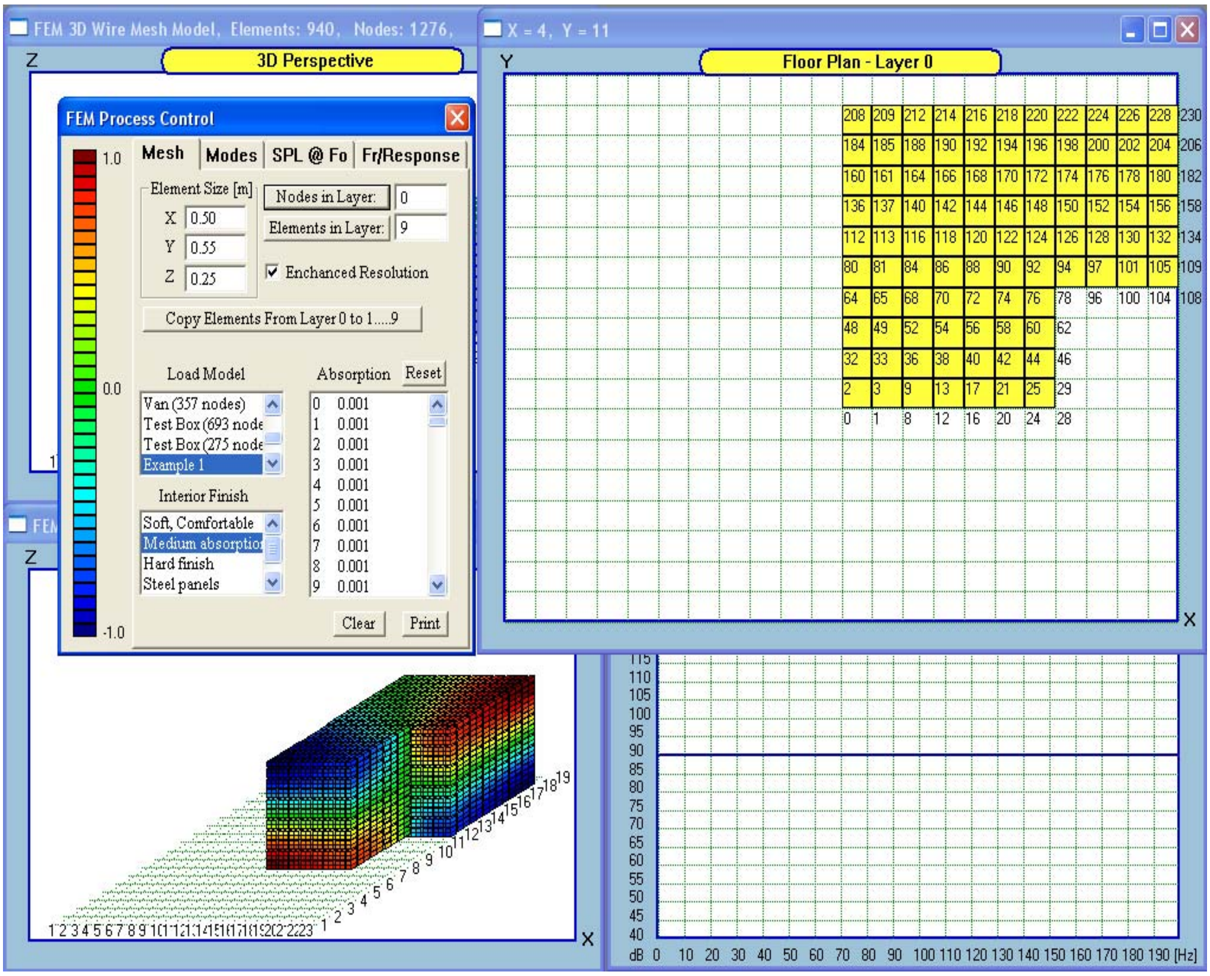


Figure 15.1 Example of a floor plan.

That is, if you avoid a pressure null at one frequency, you are likely to sit in a pressure null of another frequency. The problem here could be therefore stated in terms of what frequency range is my favorite one, so I can preserve it and which one I can sacrifice. That is, if you avoid a pressure null at one frequency, you are likely to sit in a pressure null of another frequency. The problem here could be therefore stated in terms of what frequency range is my favorite one, so I can preserve it and which one I can sacrifice.

As it is implemented, the Finite Element Method is a sophisticated tool for solving room acoustics problems. The analysis includes:

1. Creating geometrical mesh, representing the interior of the listening room. The FEM is ideally suited for complex shapes of the room.
2. Finding room natural frequencies – modes.
3. Plotting SPL distribution within the room for modes. The room can be "sliced" horizontally, so you can view the SPL of each node.
4. Finding SPL distribution for any frequency.
5. Plotting Speaker – Microphone room transfer function. Four speakers are allowed, connected "in-phase" or "out-of-phase". This effectively gives you the analysis of two dipole woofers in room.

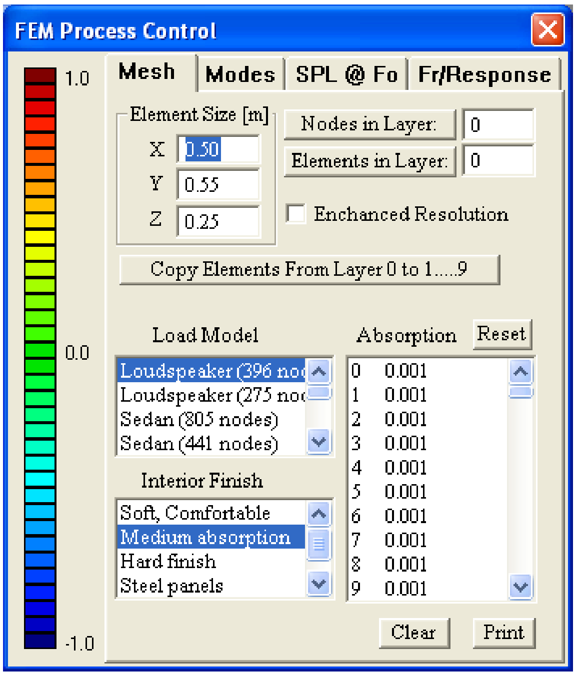



Figure 15.2. Mesh Control Tab

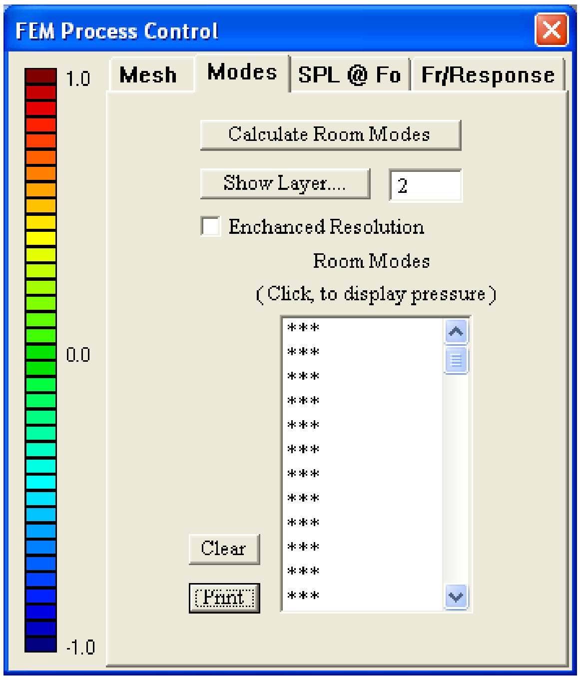


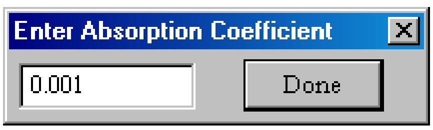

Figure 15.3. Modal Calculator Tab

**Mesh Tab**

You should start the FEM process by measuring your listening room and making a simple sketch of the floor plan. This will be helpful in determining the "brick element" dimensions, that you will use to "fill-in" the interior of the listening room – process called "meshing". Smaller brick element allows you to more accurately approximate the interior and model SPL at higher frequencies more accurately, however, there is a **limit of 1350 element imposed**, so you can not exceed this number of element in your analysis. Also, please be aware, that calculation time increases very dramatically, as you increase number of elements. The brick element dimensions are entered from **"Element Size X/Y/Z"** data fields. You can start entering mesh parameters by loading any of the built-in models (**Load Model** list box) and expanding from there, OR you can start with elements in "Layer 0" , then elements in "Layer 1", and so on, being entered into the **"Floor Plan**" window. You will see immediately the 3D structure being built on the "**3D Perspective**" on the left.

1. In the "Floor Plan" window, position your mouse pointer above the square you want to include in the floor plan – see Figure 15.4.
2. Click the Left Mouse Button – the selected rectangle will be highlighted in yellow.
3. Continue step 1,2 until you fill completely the floor plan **Layer 0**.
4. Enter "1" next to "**Elements in Layer**" button data field and press the button. This will switch the editing "Floor Plan" window to Layer 1. Go to Step 1 and enter the floor plan. If your room has parallel walls, that means ALL layers will be the same. You can save yourself editing all upper layers by clicking on "Copy Elements From Layer 0 to 1……9". This action will fill all layers above layer 0 with the same pattern. See dialogue box below.
5. **Erasing elements** from the mesh is accomplished by (1) selecting the element layer and (2) pressing the Right Mouse Button above the element you wish to erase.

There are 10 layers of elements allowed (therefore, a total of 11 nodal layers), stacked on the top of each other to approximate your listening space. Simply enter layer number and click on **"Element in Layer**" button to edit this particular layer of nodes. You can review node numbers in each layer by entering nodal layer number and pressing the "Nodes in Layer" button. If your room happens to be rectangular, you can simply copy the bottom element layer to all other layers by pressing "**Copy Elements from Layer 0 to 1…9**" button. Average absorption within the room can be controlled by selecting "Interior Finish" option from the provided list box.



Individual node absorption can be edited from the “Absorption” list box. Simply DOUBLECLICK left mouse button on the desired node and edit the coefficient. With the help of Absorption Coefficient, you will be able to approximate opened windows, carpets, heavy drapes and many other objects made from materials of known absorption. The absorbing area is determined by the location and the number of adjacent nodes with nominated Absorption Coefficients. As a refresher, in the next paragraph, we discuss some basic concepts of Absorption Coefficient. The **“Enhanced resolution”** button causes the SPL variations being displayed in finer mesh.

**Modes Tab**

You can calculate room modes by pressing the **“Calculate Room Modes”** button. When the process is complete, the modes below 150Hz will be listed in the provided list box. Click on any of the modal frequencies in the list box to have the corresponding SPL distribution displayed in the “3D Perspective – Solid Model” window. You can also review the SPL distribution layer-by-layer. Simply enter layer number and press **“Show Layer”** button

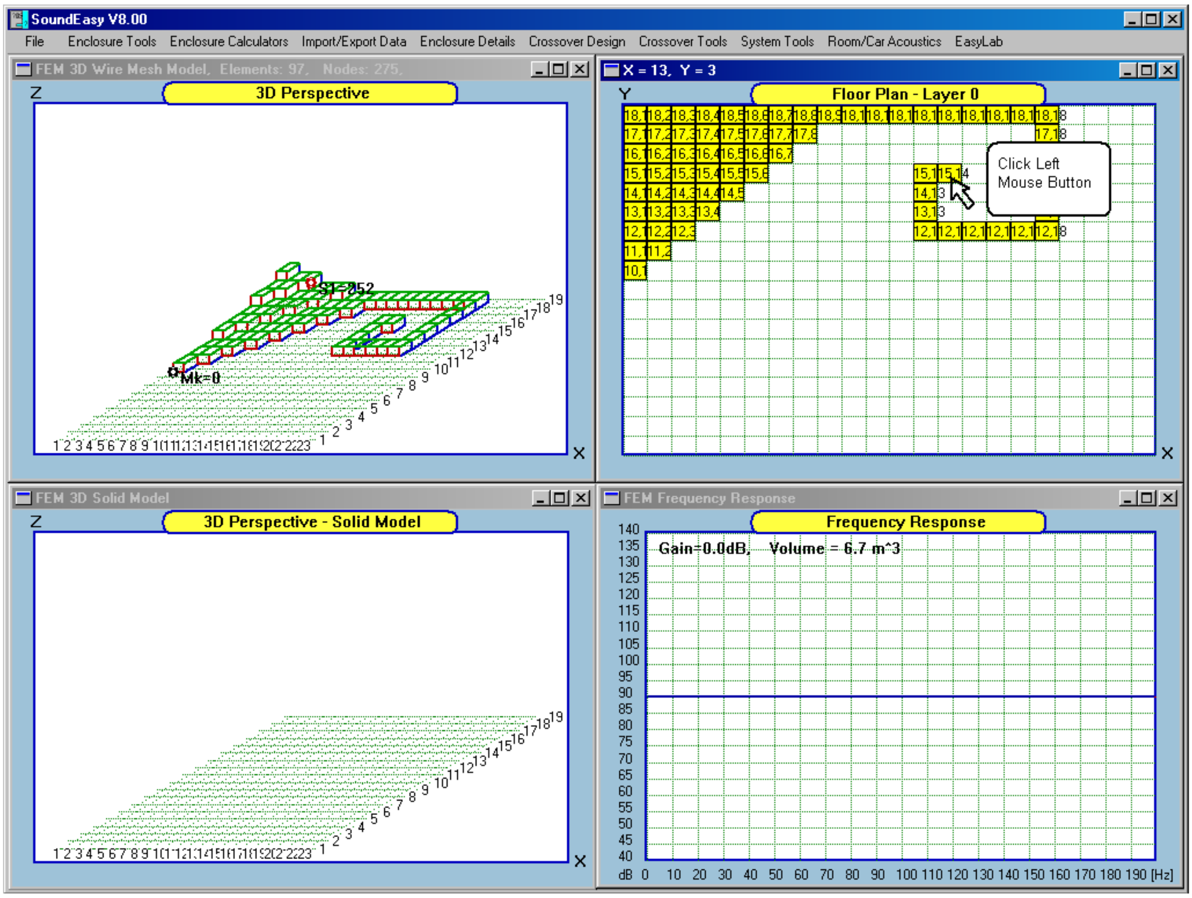



Figure 15.4. Example of building the FEM mesh model.

**SPL @ Fo Tab**

Sometimes, it may be beneficial to calculate SPL at frequencies other than modal frequencies. You can accomplish this using the “SPL @ Fo” tab. There are only two editable fields: frequency of interest (in Hz) and layer to be displayed after the SPL distribution was calculated.

**Frequency Response Tab**

The Tab has the following controls:

1. **Speaker 1 Node** - This is an editable field and is used to enter nodal location of Speaker 1. You can review node numbering by going to the “Mesh” tab. The editable field next to the “Nodes in Layer” button allows you to specify layer number you wish to inspect for nodes. Negative sign in front of the number indicates reversed phase.

2. **Speaker 2/3/4/5/6/7/8/9 Node** - Same as above. If you have only one speaker, you must enter '*' (asterisk) in this field. Negative sign in front of the number indicates reversed phase.
3. **Mike Node** - This is an editable field and is used to enter nodal location of the Listener. Again, you should review numbering of nodes as described in bullet 1.
4. **Calculate Frequency Response** - Pressing this button will activate the algorithm. The process can be quite lengthy (2-3hours), particularly for larger number of nodes. Plotted frequency response represents acoustic pressure developed within the room as generated by up to 9 speakers.
5. **Enhanced Resolution** - This check box allows you to double the resolution of the mesh approximating the listening room.
6. **Simulate** group of options:
   **Flat** = No other components are included
   **My Room** = SPL in the room with last calculated T/S-type enclosure SPL superimposed.
   **My Woofer** = SPL in the room with woofer transfer function superimposed.
   **My System** = SPL in the room with complete system transfer function superimposed.
   **My Enclosure** = Select to calculate Driving Point Impedance of your speaker enclosure. <u>This option MUST be selected for FEM modal analysis of your loudspeaker enclosure.</u>
   **Log Scale** = Switch to logarithmic scale.
7. **Clear** - Simple clears the display.
8. **Print** - Button invokes printing dialogue box.
9. **Abort** – Simply aborts plotting.
10. **Delay** – You can add small delay to each loudspeaker source.

<u>Note: Frequency response range for FEM analysis is selected from Preferences screen same as Box range.</u>

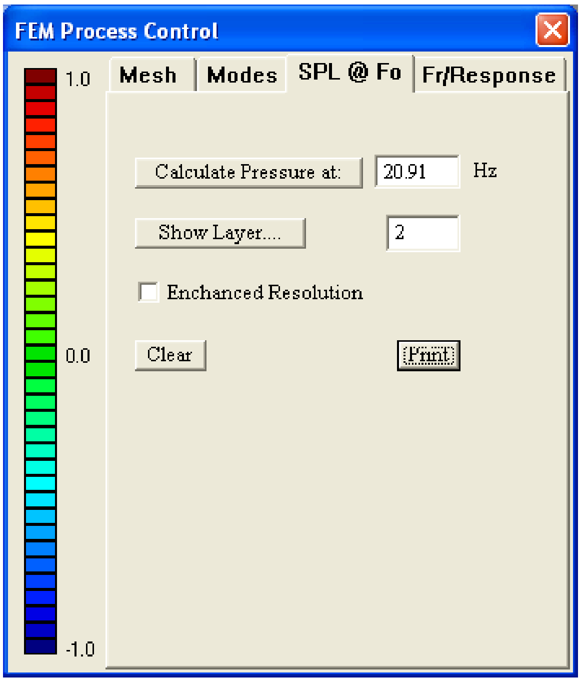



Figure 15.5. SPL @ frequency Fo Calculator

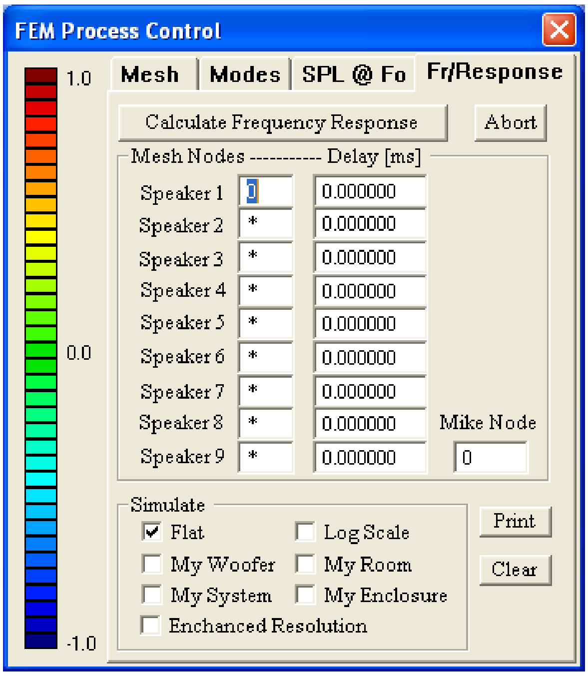



Figure 15.6 Frequency response from Nodes to Mike

The object being analyzed by the FEM screen can be sliced vertically into 10 layers: Layer 0 to Layer 9. Each layer can be entered separately, so that overall vertical shape of the object can be very complex indeed. Selection of the currently edited layer is accomplished from the control box shown on Figure 15.2. It is worth noting the numbers associated with each group of elements. The numbers suggest the flow of the activities needed to be performed sequentially.

1. **Assemble Elements in Each Layer**. There is a data field associated with the "Elements in Layer" button. This field controls which layer is currently being edited. Please enter numbers from 0 ( for Element Layer 0 ) to 9 ( for editing Element Layer 9) into this field. After entering the number press on the "Show Elements in Layer" button. If nothing was entered into this layer yet, you will see clean floor plan layer. In case, of the room under analysis being rectangular, that it the shape of the floor plan extends right up to the ceiling, please enter only Element Layer 0 shape and then use "Copy Layer 0 to 1…9" button to fill all 9 remaining element layers with the same shape as the bottom one.

2. **Review Nodes in Each Layer.** The associated button, "Nodes in Layer" will display node numbers of each of the 11 Node Layers **(please note, that there is 10 Element Layers and 11 Node Layers)**. The Layer number is entered from the associated data field.

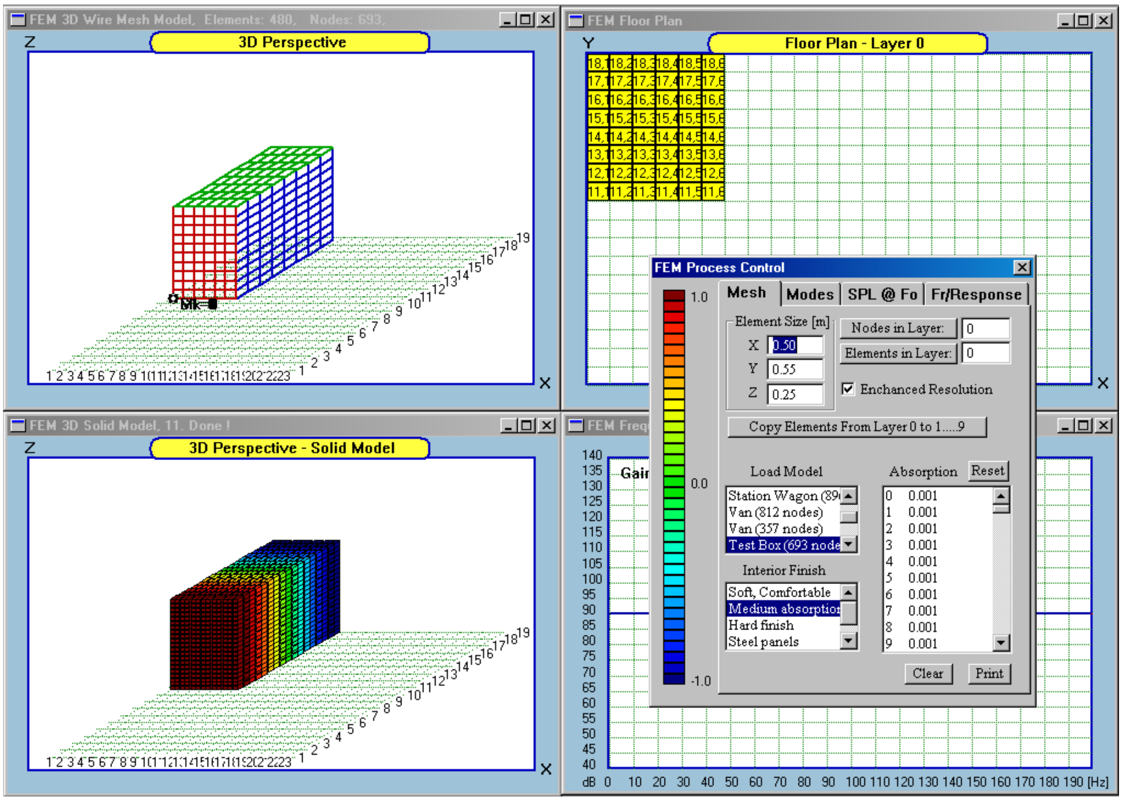



Figure 15.7. Example of modal calculation.

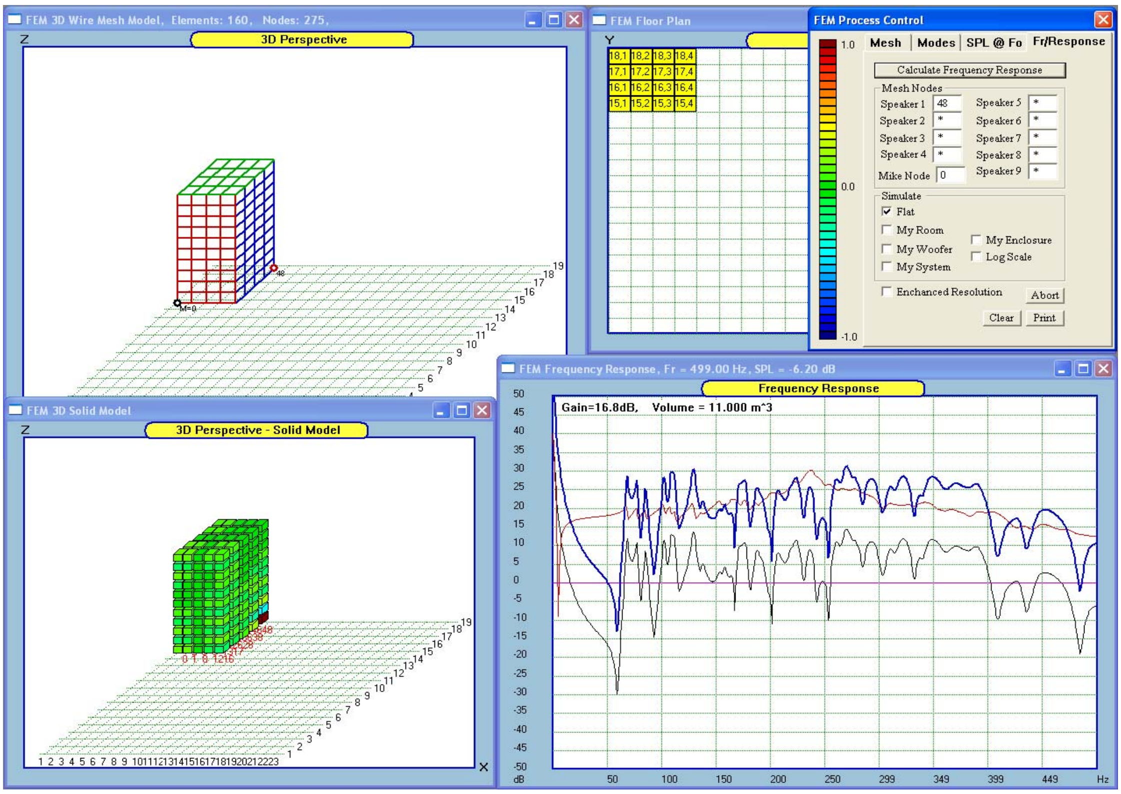



Figure 15.8. Example of frequency response plots

3. **Resonance Modes**. Calculate Room Modes is the next step in the process of analyzing the room acoustics. The FEM process is often quite long if you deal with large number of nodes and slower computer. When it has finished, you can review the whole object layer-by-layer using “**Show Layer**” button and entering Element Layer number into the associated field.

4. **Pressure From Active Sources**. Knowing the room modes enables you to make decision about placing one or two sound sources and selecting the test frequency. Then press the “**Calculate Pressure at**:” button. When the process is completed, you can review individual Element Layers by pressing the “**Show Layer**” button.

5. Finally, if the finer mesh is required, please use “Enhanced Resolution” option.

### Listening Room example

For the purpose of further analysis, we selected a listening room having the shape indicated in Fig. 15.1. The floor plan is bounded by the Nodes: 0,28,78,108,230 and 208 corners. The “brick element” we used to model the internal volume of the room has the following dimensions: X=0.5 meter, Y=0.55 meter and Z=0.25meter. The room is acoustically untreated and walls are 100% reflective - this would be a good approximation of a great majority of home listening environments at frequencies below 70Hz.

1. Fig. 15.9 shows pressure distribution of the 27Hz (lowest) mode.

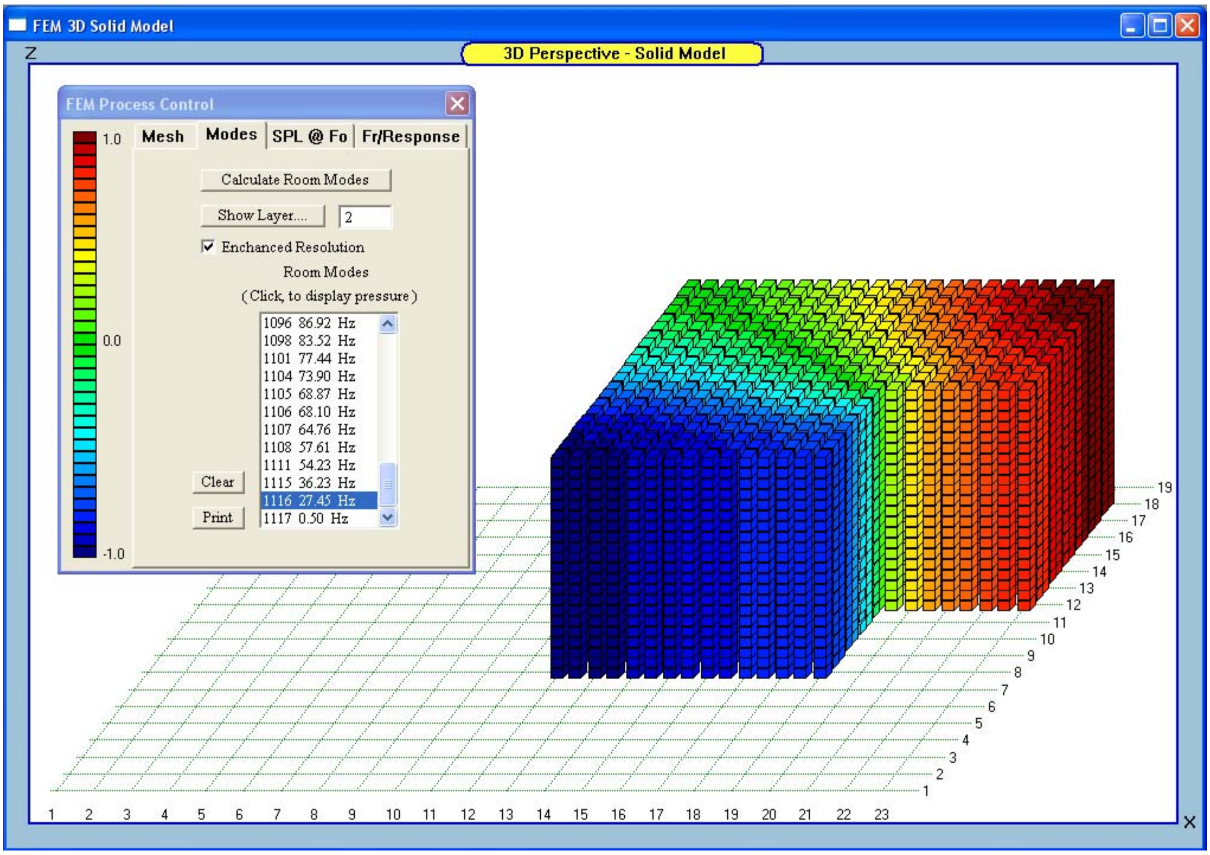



Fig. 15.9 shows pressure distribution of the 27Hz (lowest) mode

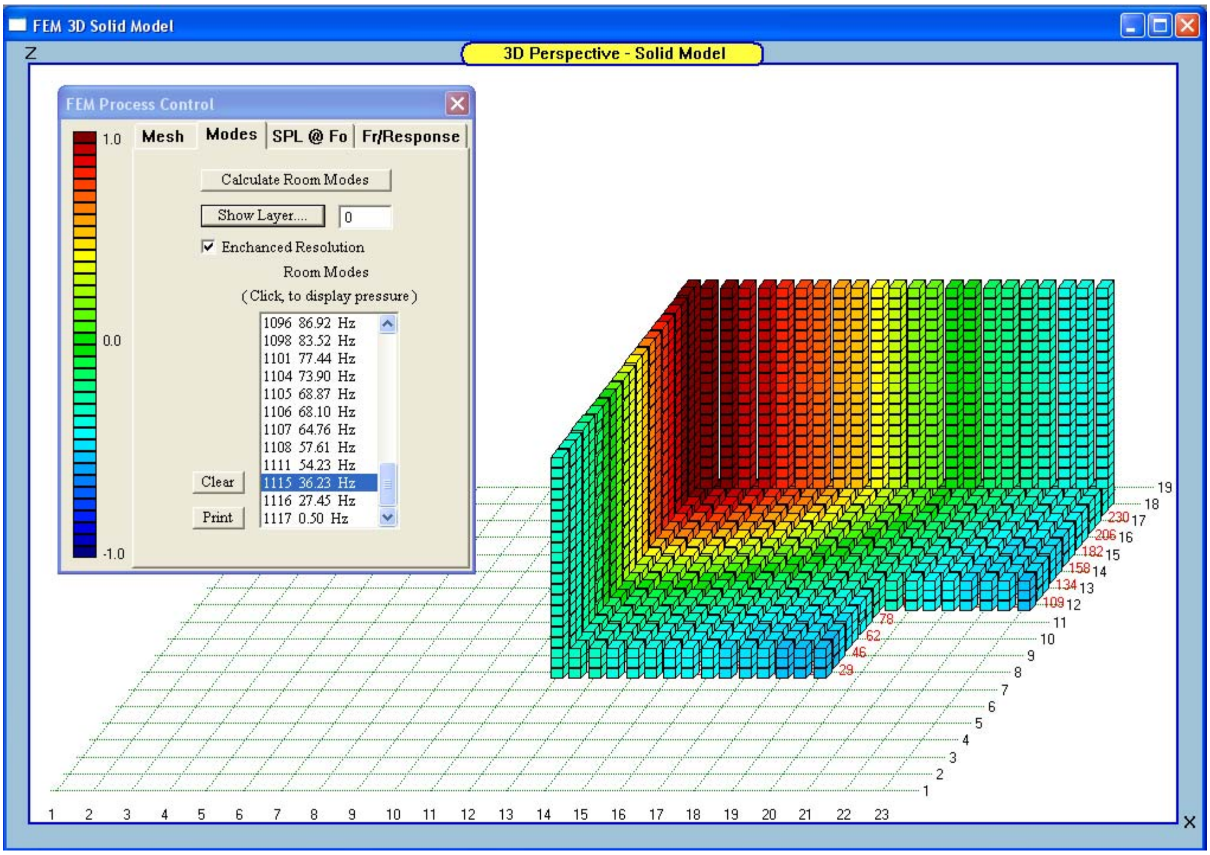



Fig. 15.10 shows pressure distribution of the 36Hz mode, showing Layer 0.

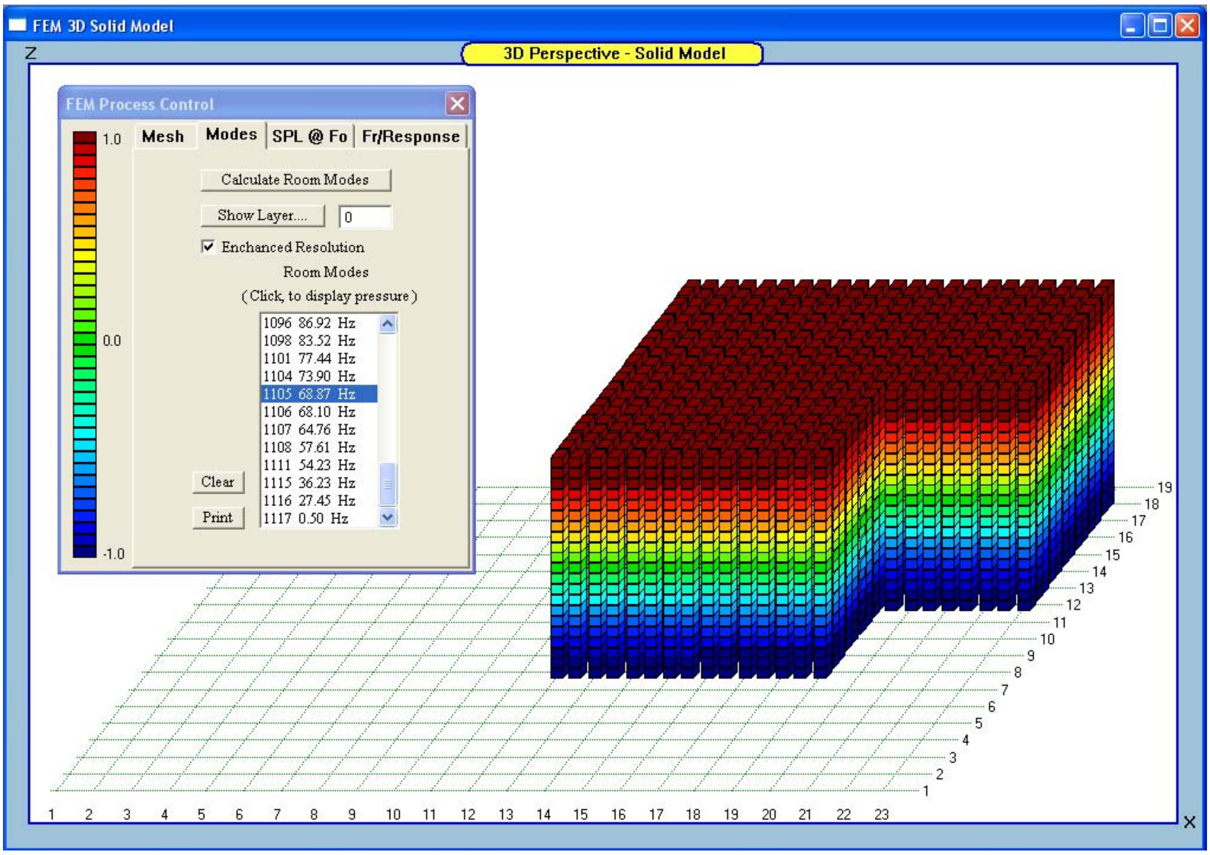



Fig. 15.11 shows 68.6Hz mode developed between the floor and ceiling

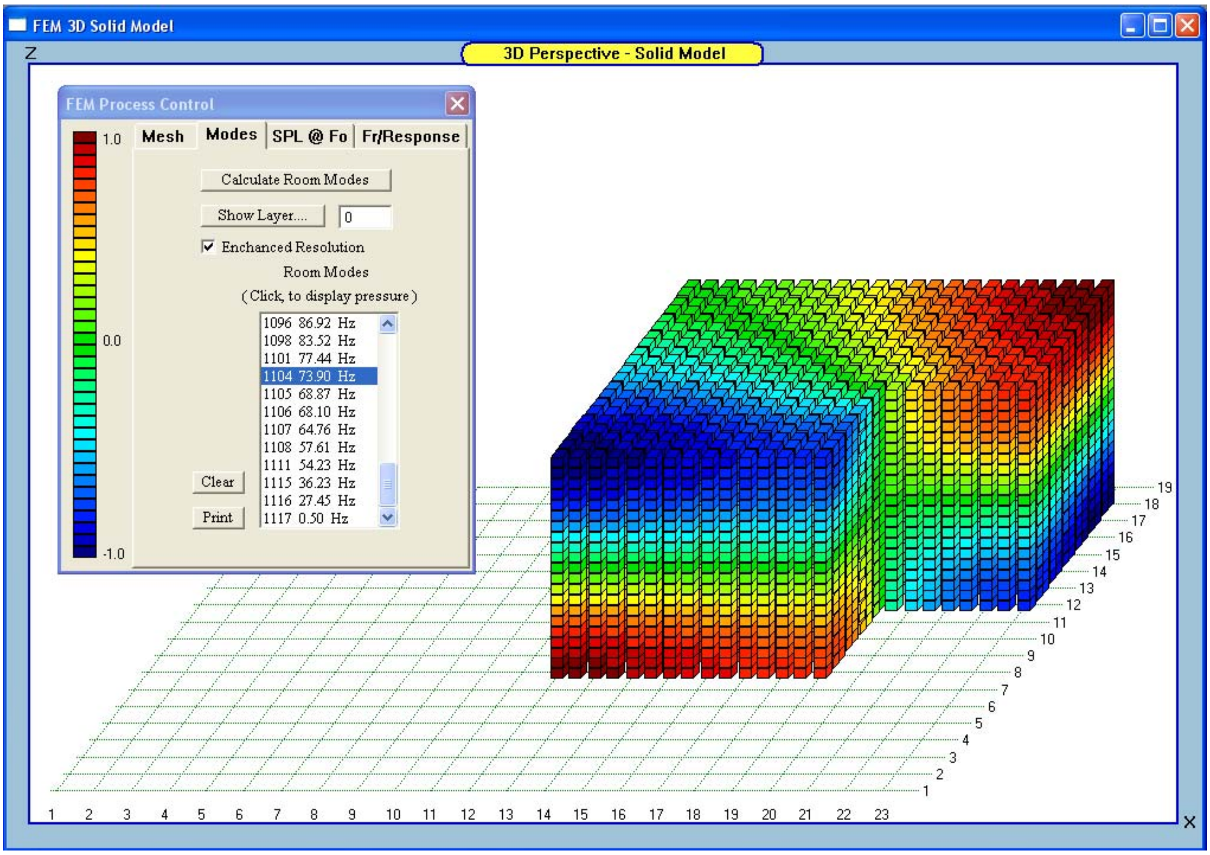



Fig. 15.12 shows tangential mode, 73.6Hz, developed between corners Nodes 78-208 and the ceiling

2. Fig. 15.10 shows pressure distribution of the 36Hz mode, which appears as a gently curved line running from X=12,Y=11 to X=18,Y=18 coordinates.
3. Fig. 15.11 shows 68.6Hz mode developed between the floor and ceiling.
4. Fig. 15.12 shows tangential mode, 73.6Hz, developed between corners Nodes 78-208 and the ceiling.

As mentioned before, higher modes, for this particular type and size of modeling elements, tend to attract a small percentage error. You can reduce the error and increase overall accuracy by selecting elements of smaller size. For the given room dimensions, the lowest mode is 27Hz and it can only develop if you place the loudspeaker in corner Node 230 and select your listening position in corner Node 0 (see Fig. 15.1). The area to avoid for listening position is the straight line between corners Node 78-208. The second lowest mode is 36Hz (see Fig. 15.10) and this one is not readily predicted without the FEM method. To energize this mode, one would place the second loudspeaker right in the Node 208 corner and avoid listening positions half-way along the walls Nodes 208+230 and Nodes 0+208 and also middle of the room. Two things need to be emphasized strongly here: (1) by placing your loudspeakers in the corners of the room, you take advantage of what is known as “room gain”. It can add as much as 10dB at low end frequencies to your loudspeaker output, as compared with the “free space” response, (2) room modes, being based on standing waves phenomenon, also amplify the sound at specific frequencies. Therefore, you do not need to locate your listening position at nodes with 100% pressure. We would strongly suggest aiming for 40-60% pressure nodes locations but avoid nodes of 0% pressure at all costs. With the above in mind, you will find that placing the subwoofers in corners Node 208 and 230 and selecting you listening area framed by [X13,Y8], [X17,Y8], [X13,Y10], [X17,Y10] points secures reasonable (not perfect) reproduction of the lowest two modes. As you can see, one would recommend a satellite system with two subwoofers and two satellites, one located in corner Node 208 and the other half-way along the Nodes 208+230 wall. The X,Y and Z nodes can be easily translated into physical space coordinates of the room with the help of the element XYZ dimensions. Higher modes, shown in Figures 15.10, 15.11 and 15.12 are useful in understanding what is the cost of this particular arrangement and we suggest you draw your own conclusions here. Having reviewed the modes, it should become easier to see, why the acoustically untreated room is a compromise. On the positive side however, you can now make an intelligent choice and understand tradeoffs between choosing this particular positioning of loudspeakers and listening area. Some of us will be tempted to take advantage of the room modes.

Say that you are interested, above all in reproducing 20Hz wavelengths - because you purchased these special woofers and designed this unique enclosure and so on. In our opinion, this is a quite legitimate point of view. Today's electric bass guitars have 5 strings and easily generate fundamental frequency of 20-30Hz. Special sound effects recorded on digital video discs are also rich in low frequencies. If your thinking follows along these lines, you need to perform Modal Analysis of your listening room, determine pressure distribution patterns and only then make an educated decision about your listening area.

**Introducing a source of sound**

In order to model harmonic acoustic behavior of an enclosed space with a sound source, the following equation is sufficient:

$$(K[][] - k^2 M[][] + j\omega C[][])p[] = j\rho_0 \omega v[] \qquad k = \frac{\omega}{c} \qquad \omega = 2\pi f \qquad j = \sqrt{-1}$$

Where: K[][] is called acoustic stiffness matrix, M[][] is the acoustic mass matrix, C[][] is the damping of the system, p[] is the sound pressure vector, v[] is the excitation vector in qubic meters per second, f is the test frequency and c is the speed of sound.

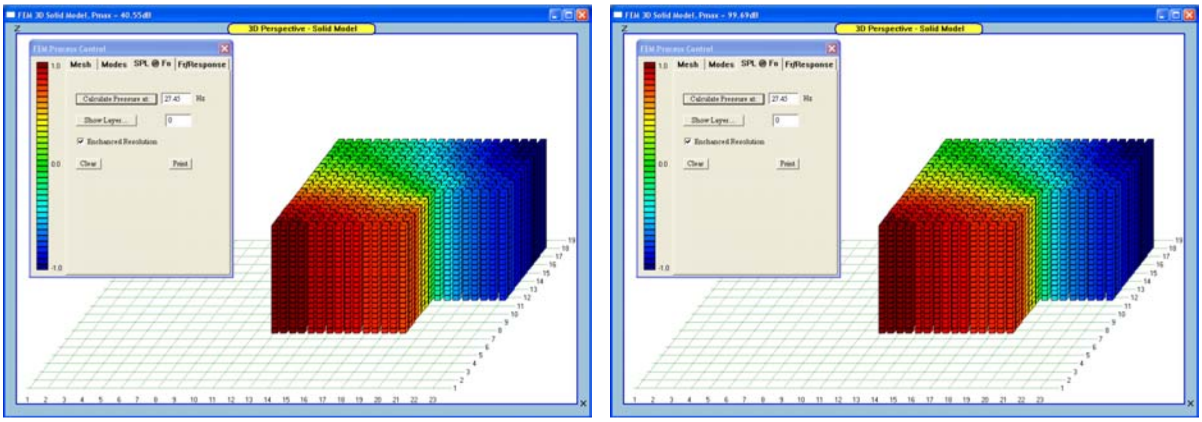


Fig 15.13 Source of sound at Node 184, **Pmax=40dB** Source of sound at Node 0, **Pmax=99dB**

Assuming C[][]=0, that is no damping in the system, the above equation simplifies to:

$$(K[][] - k^2 M[][])p[] = j\omega v[]$$

For the source of the sound I assume a "point source", which is convenient, as it can be located in any of the mesh nodes. Larger sources would have to be accounted for as a part of the room boundary. Mathematically, the problem now reduces to assembly of the K[][] and M[][] matrixes and inverting the expression in the brackets. This way, vector p[] can be found for any frequency and location of the sound source represented by excitation vector v[]. For room modes, the expression in the brackets:

$$(K[][] - k^2 M[][]) = 0$$

SoundEasy uses "brick" (8-node) elements to approximate the volume of the listening room and the functions describing the pressure distribution between the nodes are linear. This is important, as knowing the pressure at the nodes, one can calculate pressure at any distance between them. One of the limitations of the above approach is that FEM can not model the "close field" sound pressure very accurately. It would take much more dense (finer) mesh to do this job properly. We are not however concerned with this limitation, because the job at hand is to look at the whole room. For the purpose of illustrating the application of the FEM method we selected a listening room shape indicated on Figure 15.1. This is an L-shaped room with wall Nodes 0-28 slightly longer than Nodes 108-230 wall. This deliberate lack of symmetry will make the analysis perhaps more difficult, but will emphasize the usefulness of the FEM.

**Source at room nodal line**

We can now keep the test frequency at the first room mode and move the source to the pressure null line of the first mode. This situation is depicted on Figure 15.13 and it is easily observable, that despite radiating the exact room modal frequency, the source fails to **fully energize** the room mode

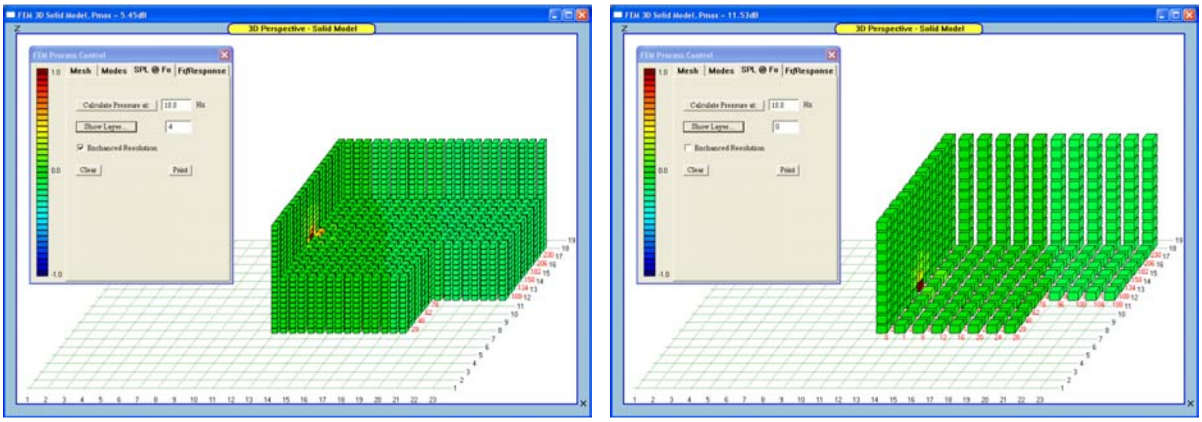


**Fig 15.14 Source of sound at Node 620 and 18.8Hz, Pmax=5.45dB, Node 80, Pmax=11.45dB**

Consider the following sequence: (1) If the "point source" radiates in free space, it would produce some sound pressure, which I mark as "unity". (2) When this source is placed in front of a hard wall (half-space) the sound pressure is reinforced by the reflections from the wall. (3) If in addition, the source is moved to the wall-floor junction, the resulting sound pressure is further reinforced and finally, (4) if the source of the sound is moved into the corner of the room, the sound pressure is up by some 16-18dB. Now, (5) if the frequency of the sound source happens to be one of the room resonant modes, the sound pressure at the source is dramatically magnified by the resonant effect and lack of dumping. ( We assumed matrix C[][]=0 ). The display is arranged such a way, that nodal pressure is normalized to the maximum pressure within the room. This maximum pressure is marked "0.0dB", so all other nodes will exhibit negative pressure values, as shown on the color coded "Pressure Map" on each screen dump. **The actual level of the "Pmax=" is displayed in windows caption bar, relative to sound pressure radiated from a "point source" in free space**. Generally, if the Pmax were above 80dB mark, you would be approaching a resonant mode. When the Pmax exceeds 80dB you hit the mode. Level of 40dB indicate, that the source fails to fully energize the mode.

Levels below 18dB relate to various locations of the source at non-modal frequencies. In general, sound field of the complex-shape enclosure can be modeled for any location of the source or sources of the sound. For the non-modal frequencies, the resulting sound field typically exhibits variations near the source and a gradual increase in intensity toward opposite wall. Please note, that computer rounding errors also contribute to the value of Pmax at modal frequency and finally, the modal frequencies, as given in the previous article, are calculated and rounded to the second decimal point, as in "real life", there is no need for greater accuracy.

**At a flat wall location, the Pmax= parameter is close to a typical wall-floor value of 6dB. The source was located at Node 620. At Node 80, (wall/floor joint) Pmax equals approximately 12dB. Finally, at the corner location, the Pmax=17.84dB, again, approximately the theoretical 18 dB for corner location**.

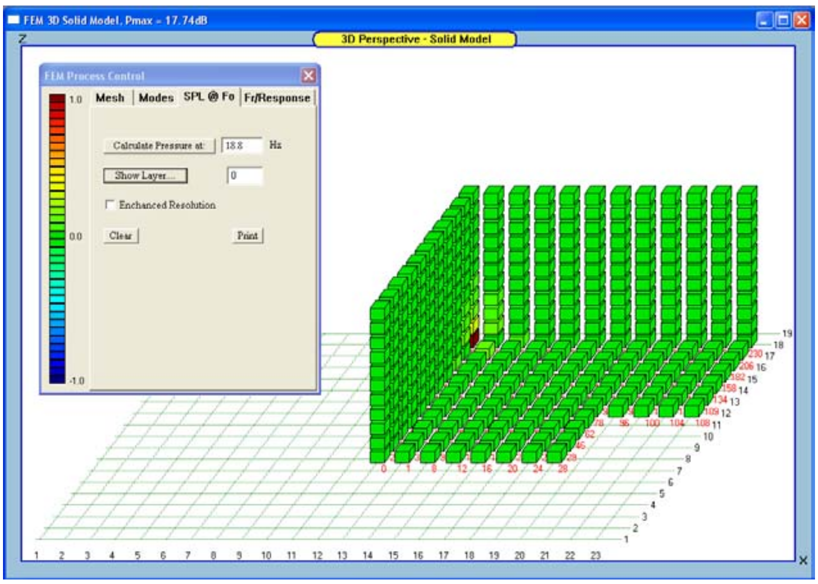


**Figure 15.15. Corner location and non-modal frequency, Pmax=18dB**.

**3D Delaunay FEM Screen**

**Method of Analysis**

Architectural style of today reflects our relaxed lifestyle. As a result of this, open layout of several interconnecting rooms or areas is very common. This type of dwelling allows the air and sound to freely flow from one room to another (no doors) and as such, one large, complex shape cavity is created. An example of such complex cavity is shown above on Figure 15.16. Here we have a floor plan of a house comprising of an entry hall, sitting room to the left, formal dining room to the right, and kitchen and living room further down the hall. **Our task is to obtain lowest (<100Hz) modal frequencies of the dwelling and pressure pattern distribution for those frequencies**. For the purpose of analysis, it was assumed that walls absorb negligible amount of acoustic energy. The longest distance between the most distant walls is typically much larger than the distance between floor and ceiling.

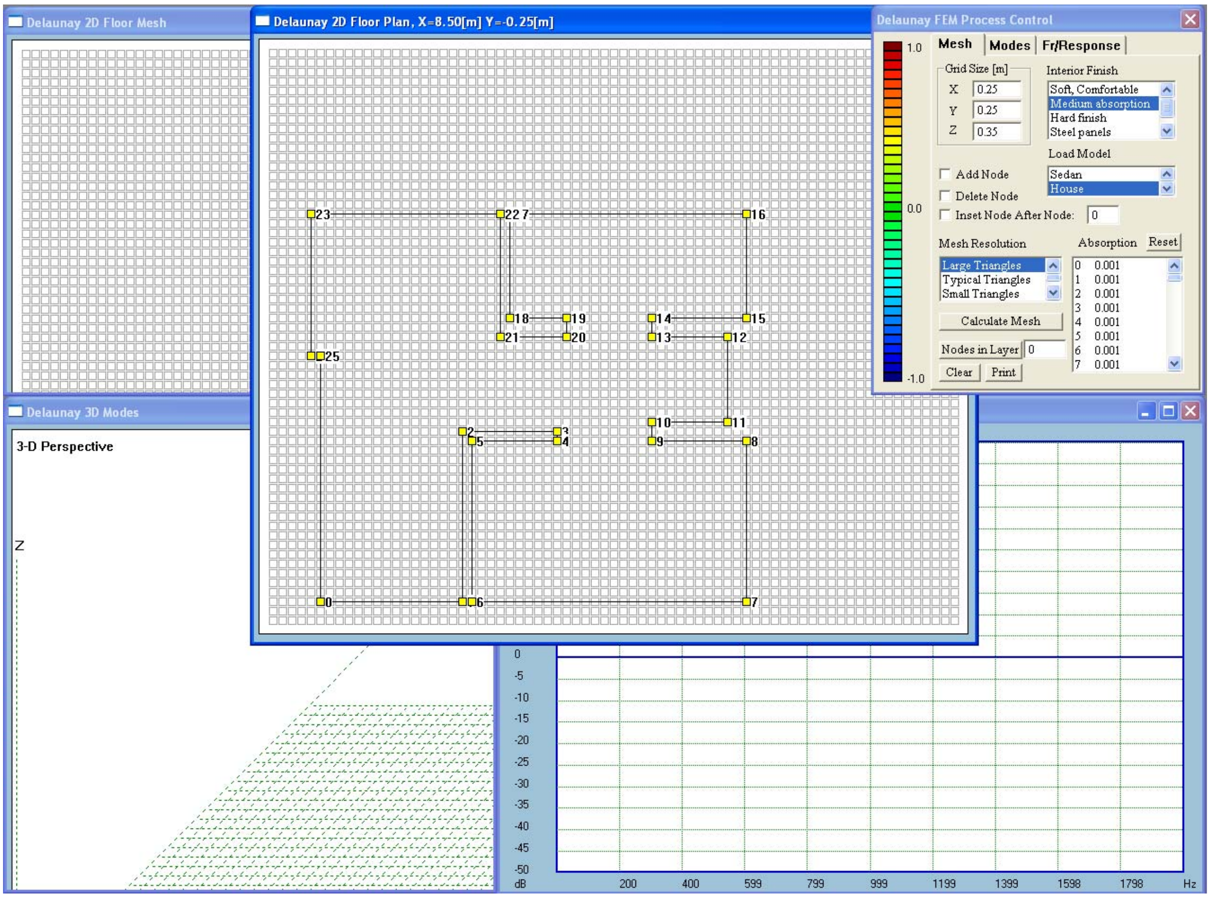


Fig 15.16 . An example of such complex cavity is shown above

Therefore the lowest modes will be tangential modes - these are modes NOT involving floor or ceiling reflections. In order to calculate the height of the element, you need to **measure the floor-wall distance and divide it by 6**. For example: distance between floor and wall is 3.30 meter. Therefore the height of the "wedge" element is 3.30/6 = 0.55 meter. The smallest increments (resolution) along the X and Y axes is also determined by the user.

This way, a large and **complex floor plan may include thin walls** in the model - this feature is essential in modeling complex floor plan of the house. The Delaunay Triangulation process is fully automated and only requires that the you provide the floor plan as the input. This is accomplished by drawing the perimeter of the area being analyzed as a **Planar Straight Line Graphic (PSLG)**. The graph must be entered in an **anticlockwise** direction. Please select the "3D Delaunay Screen" from the main menu.

The steps are described below:

1. Position mouse arrow pointer on the drawing right above the first or "start" node to be marked and CLICK the LEFT mouse button. The selected "start" node will be marked with a number when the button is clicked.

2. Move the mouse arrow pointer to the **right of the start node**, where you wish the "end" node of the line should be and CLICK again the LEFT mouse button. The line will be drawn during mouse movements and when the button is released, the "end" node will be marked, and numbered. **It is important to "orient" the SLPG anticlockwise right at the beginning, by positioning the second node to the right of the first node.** At this moment, you have one straight line with two nodes drawn on the plotting area. This represents a wall in your floor plan.

3. Position the mouse arrow pointer where you want to place the next node of the graph and CLICK the LEFT mouse button again. The node will be marked with a number and you are have the second "wall".

4. Continue to move the mouse arrow pointer to where you wish the consecutive nodes to be placed and CLICK on every node. Lines will be drawn automatically and nodes will be marked, and numbered.

5. Continue drawing walls until the closed, bounded floor plan is completed.

**When drawing SLPG, you must remember the following: The graph MUST be oriented anticlockwise and MUST be closed - that is the "end" node of the last wall MUST be "start" node of the whole SLPG.**

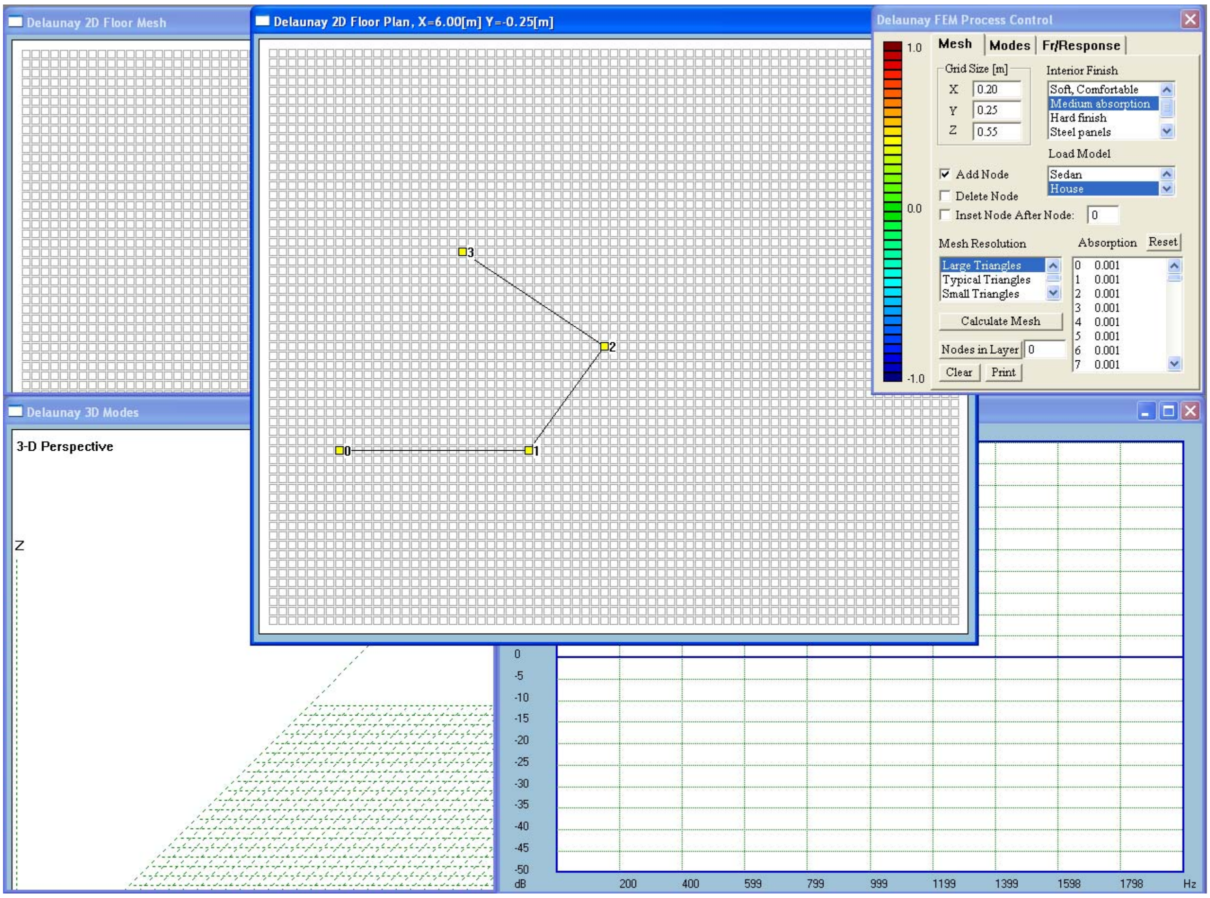



Fig 15.17 shows the first three walls entered.

**How it is done - Vibration basics**

One of the types most common types of analysis for structures are: (1) the natural frequency analysis or eigenvalue analysis and (2) steady state response of the structure to a harmonic force at a selected frequency. We are going to apply both method to analyze a "listening room" acoustics. The general equation of motion describes the balance required for all inertial and damping forces in the equation.

$$M[][] * \frac{\partial^2 D[]}{\partial t^2} + C[][] * \frac{\partial D[]}{\partial t} + K[][] * D[] = F[]$$

Where M[][] represents the structure mass matrix,
C[][] is the structure damping matrix,
K[][] is the structure stiffness matrix,
F[] is time varying nodal load vector and
D[] is the nodal displacement vector with its acceleration and velocity.

**Eigenvalue Analysis**

Assuming, that all nodes vibrate in a sinusoidal manner with a peak displacement P, we can write:

$$D[] = P * \sin(\omega t)$$

$$\frac{\partial D[]}{\partial t} = P\omega \cos(\omega wt)$$

$$\frac{\partial^2 D[]}{\partial t^2} = -P\omega^2 \sin(\omega t) \qquad \omega = 2\pi f \qquad j = \sqrt{-1}$$

Also, we shall assume no damping of the structure, which implies C[][] = 0 and no external forces, so that F[] = 0 as well. The general equation of motion simplifies now to:

The above equation can now be solved as a standard eigenvelue and eigenvector problem.

$$(K[][] - k^2 M[][])P[] = 0 \qquad k = \frac{\omega}{c}$$

Theoretically, we imply that if the structure is deformed into a mode and then released, it would continue to vibrate indefinitely due to the absence of damping. However, practical structures always have some damping, so that the vibrations will eventually decay.

**Frequency Response Analysis**

In order to model harmonic acoustic behavior of an enclosed space with a sound source, the following equation is sufficient:

$$(K[][] - k^2 M[][] + j\omega C[][])p[] = j\rho_0 \omega v[]$$

Where: K[][] is called acoustic stiffness matrix,
M[][] is the acoustic mass matrix,
C[][] is the damping of the system,
p[] is the sound pressure vector,
v[] is the excitation vector in cubic meters per second,
f is the test frequency and
c is the speed of sound.

For the source of the sound one could a "point source", which is convenient, as it can be located in any of the mesh nodes. Larger sources would have to be accounted for as a part of the room boundary. Mathematically, the problem now reduces to assembly of the K[][] and M[][] matrixes and inverting the expression in the brackets. This way, vector p[] can be found for any frequency and location of the sound source represented by excitation vector v[]. Solving this equation for different discrete input frequencies determines the frequency response of the structure. When the test frequencies are equal to the natural modes, the solution algorithm fails due to division by zero.

**Stiffness and Inertia Matrixes**

Derivation of stiffness and inertia matrixes is beyond the scope of this article, however, I would like to describe briefly the elements involved. Our interest will focus on a volume V enclosed by a surface S. The surface is acoustically hard, so that normal velocity on the surface is zero. Under those conditions the pressure p, must satisfy the wave equation

$$\nabla^2 p + \left(\frac{\omega^2}{c^2}\right)p = 0 \qquad\qquad \frac{\partial p}{\partial n} = 0$$

Variational principle, equivalent to the above equations can be prescribed as follows:

$$\delta\int_V \frac{1}{2}\left[(\nabla p)^2 - \left(\omega^2 / c^2\right)p^2\right]dV = 0$$

In order to maintain continues pressure between the nodes, an 8-node “brick” elements can be used to fill the volume V. Elements k[][] of the stiffness matrix K[][] can be expressed as follows:

$$k_i[][] = \int_{Vi} B_i^T[][]B_i[][]dV$$

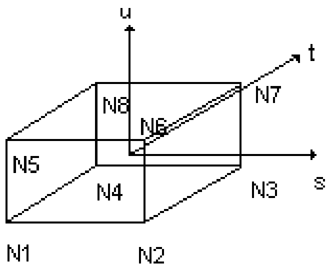
Where:

$$B_i = \begin{bmatrix} \dfrac{\partial N_i}{\partial x} \\ \dfrac{\partial N_i}{\partial y} \\ \dfrac{\partial N_i}{\partial z} \end{bmatrix}$$

And the elements m[][] of the inertia matrix M[][] is expressed as follows:

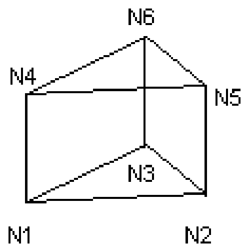
$$m_i[][] = \int_{Vi} N_i^T[]N_i[]dV$$

Both elemental matrixes for 8-node element are 8x8 in size. Vector N[] represents eight linear shape functions defined as below:




N1 = (1 - s)(1 - t)(1 - u)/8 N2 = (1 + s)(1 – t)(1 – u)/8
N3 = (1 + s)(1 + t)(1 – u)/8 N4 = (1 – s)(1 + t)(1 – u)/8
N5 = (1 – s)(1 – t)(1 + u)/8 N6 = (1 + s)(1 – t)(1+ u)/8
N7 = (1 + s)(1 + t)(1 + u)/8 N8 = (1 – s)(1 + t)(1 + u)/8

For the 6-node "wedge" element, vector N[] represents six linear shape functions defined as below:



N1 = s(1 - u)/2 N2 = t(1 – u)/2
N3 = (1 - s - t)(1 – u)/2 N4 = s(1 + u)/2
N5 = t(1 + u)/2 N6 = (1 - s - t)(1+ u)/2

With the above in mind, the variational principle can be expressed in matrix form as:

$$(K[][] - k^2 M[][])P[] = 0 \qquad k = \frac{\omega}{c}$$

Due to complexity of the resulting expressions for element k[][] and m[][] they are best evaluated by using numerical integration techniques. Integration can be facilitated by Gauss integration scheme, requiring integration limits of +1 and –1. We can change integration from x,y,z space to s,t,u space with the help of Jacobian J[][] of the transformation resulting immediately in the volume element dV being expressed as follows:

$$dV = |J|dsdtdu$$

Where |J| is the determinant of Jacobian matrix J[][].

**Jacobian of the Transformation**

$$\begin{bmatrix} \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \\ \frac{\partial N_i}{\partial u} \end{bmatrix} = [J] \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix}$$

Where J[] is a ***Jacobian*** of the transformation expressed as follows:

$$[J] = \begin{bmatrix} \frac{\partial N_1}{\partial s} & \frac{\partial N_2}{\partial s} & \frac{\partial N_3}{\partial s} & \frac{\partial N_4}{\partial s} & \frac{\partial N_5}{\partial s} & \frac{\partial N_6}{\partial s} & \frac{\partial N_7}{\partial s} & \frac{\partial N_8}{\partial s} \\ \frac{\partial N_1}{\partial t} & \frac{\partial N_2}{\partial t} & \frac{\partial N_3}{\partial t} & \frac{\partial N_4}{\partial t} & \frac{\partial N_5}{\partial t} & \frac{\partial N_6}{\partial t} & \frac{\partial N_7}{\partial t} & \frac{\partial N_8}{\partial t} \\ \frac{\partial N_1}{\partial u} & \frac{\partial N_2}{\partial u} & \frac{\partial N_3}{\partial u} & \frac{\partial N_4}{\partial u} & \frac{\partial N_5}{\partial u} & \frac{\partial N_6}{\partial u} & \frac{\partial N_7}{\partial u} & \frac{\partial N_8}{\partial u} \end{bmatrix} \begin{bmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \\ x4 & y4 & z4 \\ x5 & y5 & z5 \\ x6 & y6 & z6 \\ x7 & y7 & z7 \\ x8 & y8 & z8 \end{bmatrix} =$$

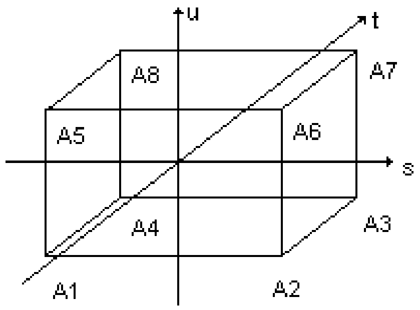
$$\begin{bmatrix} \sum_{i=1}^{8} \frac{\partial N_i}{\partial s} x_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial s} y_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial s} z_i \\ \sum_{i=1}^{8} \frac{\partial N_i}{\partial t} x_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial t} y_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial t} z_i \\ \sum_{i=1}^{8} \frac{\partial N_i}{\partial u} x_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial u} y_i & \sum_{i=1}^{8} \frac{\partial N_i}{\partial u} z_i \end{bmatrix}$$

Now, the derivatives of the shape functions in respect to x,y and z coordinates can be calculated by inverting the Jacobian matrix J[][].

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \\ \frac{\partial N_i}{\partial u} \end{bmatrix}$$

### Gaussian Integration

Example of Gaussian integration scheme for the 8-node "brick" element and N=2 is being used and the location of the points together with the associated weights is shown below.




| | Weight | s | t | u |
|---|---|---|---|---|
| A1 | 1.0 | -0.57735 | -0.57735 | -0.57735 |
| A2 | 1.0 | 0.57735 | -0.57735 | -0.57735 |
| A3 | 1.0 | 0.57735 | 0.57735 | -0.57735 |
| A4 | 1.0 | -0.57735 | 0.57735 | -0.57735 |
| A5 | 1.0 | -0.57735 | -0.57735 | 0.57735 |
| A6 | 1.0 | 0.57735 | -0.57735 | 0.57735 |
| A7 | 1.0 | 0.57735 | 0.57735 | 0.57735 |
| A8 | 1.0 | -0.57735 | 0.57735 | 0.57735 |

### Assembly of System Matrixes

**Local** nodes of individual elements are always numbered as 1 to 8 for "brick" elements and 1 to 6 for "wedge" elements. However, when the complete mesh has been generated, **global** coordinates of all nodes and global numbering scheme for all components become available, and at this point of time we know which elements share the same nodes. It is therefore possible to assign the global node numbering and the global element coordinates to the **local** nodes of each element. The process is called mapping and can sometimes be "tricky". Assembly of the stiffness K[][] and inertia M[][] system matrixes involves calculation of derivatives of the shape functions ***Ni*** and can be quite complex process.

### Solution of Eigenvalue problem

Eigenvalue problem can be solved using Hauseholder Algorithm and Tridiagonal QLI Algorithm. However, this process requires symmetrical matrixes for its input. Assuming that matrixes K[][] and M[][] are symmetric of order n, the eigenvalue problem can be stated as:

$$(K[][] - \lambda M[][])X[] = 0$$

The X[] is a vector called eigenvector and $\lambda$ is a scalar called eigenvalue. The above can be converted to standard eigenvalue problem by pre-multiplying both sides by inverted matrix M[][].

$$A[][] = M^{-1}[][] * K[][]$$

However, matrix A[][] is often non-symmetrical despite M[][] and K[][] being both symmetrical. If the method adopted for solving the eigenvalue problem requires A][] to be symmetrical, the following procedure may be used:

1. Using Choleski method, decompose M[][], so that only upper triangular U[][], matrix will be used:
2. Formulate A[][] as:

$$M[][] = U^T[][] * U[][] \qquad A[][] = (U^T[][])^{-1} * K[][] * (U^{-1}[][])$$

3. Matrix A[][] is now symmetrical and suitable for Hauseholder Algorithm and Tri-diagonal QLI Algorithms.

### Generating Mesh

Generating mesh is a large subject in its own right. Mesh is the way to translate information about the physical parameters of the object being analyzed into a set of numbers understood by the program. Vastly simplified version is presented here, however, good results can be achieved for a number of applications.

### Delaunay Triangulation Algorithm

The process commences with the **Planar Straight Line Graphic** (PSLG) description. In simple terms, the PSLG it is the closed contour (perimeter) encompassing the area to be triangulated. The coordinates (nodes) are entered as a graph oriented counterclockwise, and using mouse pointer. The next step is the Delaunay Triangulation process. It needs to be noted, that final results of the triangulation is expected to be a uniform mesh with a predetermined maximum triangle size, suitable for Finite Element Method mesh. Delaunay triangulation of a point set ***P***, is a triangulation with the property that no point in the point set ***P*** falls in the interior of the circumcircle (a circle that passes through all three vertices) of any triangle in the triangulation. Initially, the point set ***P*** is the set of nodes of the PSLG, as determined by the user.

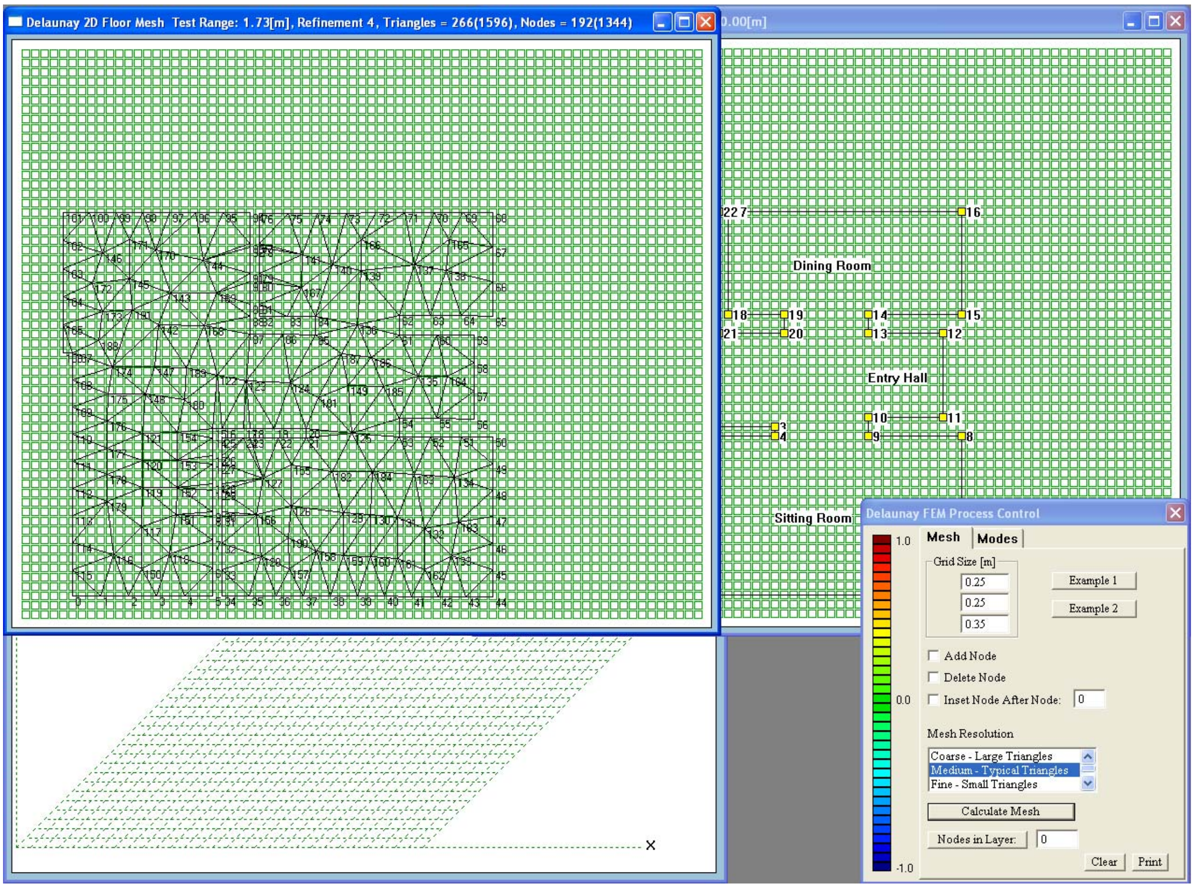


Fig 15.18 Delaunay Trangulation completed.

The point set P is then immediately expanded by splitting each edge of the PSLG into smaller sections whose length is determined by the maximum triangle size, as specified by the user. The resulting point set ***P*** is then triangulated. At this moment, there is no points inside the PSLG, therefore the first pass of the triangulation produces very long and "skinny" triangles. The record of the longest edge ***L***, is maintained for future reference. In the next loop, the longest edge is compared against the required triangle size and if a triangle with a longer side is found, the triangle is destined to be split by inserting an extra point into the offending triangle. Once all triangles are checked and all extra points are inserted, the whole point set ***Pnew*** is then re-triangulated. The process continues until there is no more extra points to be inserted. The following can be noted:

1. Each triangulation pass produces "denser" set of points from which Delaunay triangles are build. However, it is wasteful to check the whole point set ***Pnew*** for the purpose of building Delaunay triangles. Only the points located no further from the current "seed" edge than longest edge ***L***, need to be considered for potential new Delaunay triangle.
2. The algorithm maintains a list of all edges that were already used as "seeds" for constructing Delaunay triangles. These edges are never used again in a current pass.
3. The Delaunay triangulation uses **Orientation Test** - Check whether or not a point lies to the left of a vector. If point "C" lies to the left of the vector AB, the cross product of the vectors AB *AC will be larger than 0. If points "A", "B", and "C" are collinear, the cross product of these two vectors will be equal to 0. If point "C" lies to the right of the vector AB, the cross product of the vectors AB * AC will be smaller than 0.

4. Also, the Delaunay triangulation uses **In Circle Test** - Check whether or not a point lies inside the circumcircle of a triangle. The algorithm uses the InCircle Test to make sure that there is no point that is located inside the circle which passes through the three points it found. These three points can be now connected to form a Delaunay triangle.
5. Finally, the algorithm checks for "crossed edges" of triangles.

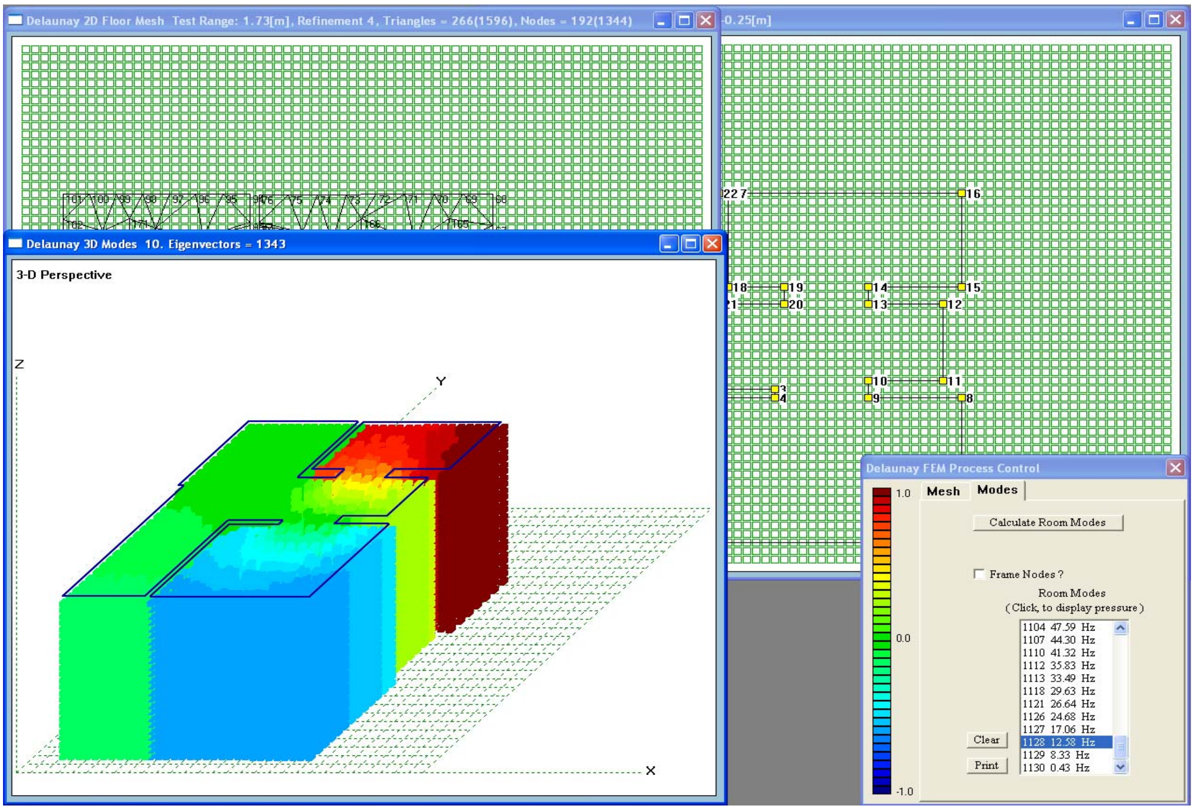


Fig 15.19 Sound field pressure distribution in the house.

Finite Element Method has been successfully implemented to accomplish the task of determining complex-shape room modes, because the method lends itself particularly well to complex geometry of the space being analyzed. All essential mathematical components of the whole analytical process have been nominated and briefly explained. Attempt to solve a problem of this nature without a computer would prove to be too formidable task to undertake, if only because of the shape complexity of the volume being analyzed. The FEM suite of tools are well suited to help you out in a situation like this. Data generated by the computer is intended to assist you in making educated decisions about the loudspeaker-listener interaction in your **individual** listening environment.

## Control Dialog Boxes

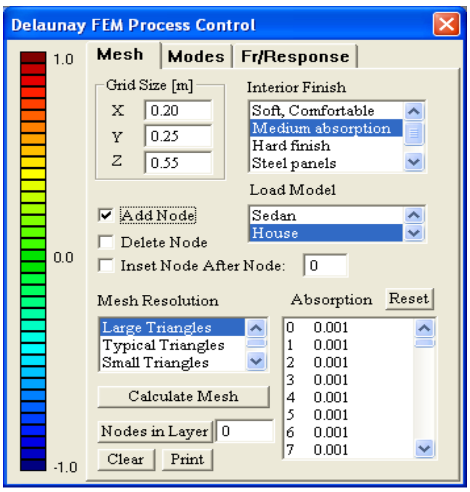


Fig 15.20 Mesh control TAB

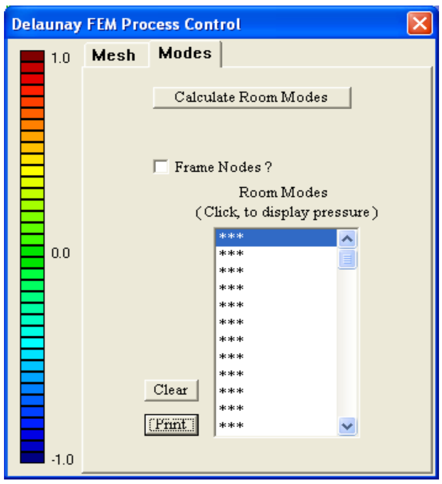


Fig 15.21 Delaunay Modes TAB.

**Delaunay Mesh TAB**

1. **Grid Size** [in meters] – X, Y, Z dimensions. Enter the required dimensions into these data fields.
2. **Load Model –** A built-in set of examples.
3. **Interior Finish –** Select the closest fit of the walls of the interior.
4. **“Add Node”** – if checked, you can add a node to the SLPG graph.
5. **“Delete Node”** – if checked, you can delete a node to the SLPG graph. Example below shows most of the internal nodes deleted from the “open plan house” example.
6. **“Insert Node After Node”** – if checked, you can insert a node to the SLPG graph. The new node will be inserted after the nominated node.
7. **Absorption** – Select absorption coefficient of individual nodes. **Reset** button included.
8. **“Mesh Resolution”** – List box to select 4 sizes used for Delaunay triangles.
9. **“Calculate Mesh”** – Press this button to generate triangular mesh of the floor plan.
10. **“Nodes in Layer”** – You can review node numbering in each of the 7 layers (0…..6).
11. **“Clear”** – Clears the mesh and SLPG graph.
12. **“Print”** – Opens printer dialogue box.

**Delaunay Modes TAB**

1. **“Calculate Room Modes”** – Once you are happy with the mesh, you can calculate room modes for this particular floor plan.
2. **“Frame Nodes”** – The resulting color-coded nodal pressure is displayed with the rectangles framed.
3. **“Room Modes”** – List box which allows you to select and display nodal pressure patterns for each calculated mode. Just click on the desired modal frequency.
3. **“Clear”** – Clears the mesh and SLPG graph.
4. **“Print”** – Opens printer dialogue box.

**Frequency Response Tab**

1. **Speaker 1 Node** - This is an editable field and is used to enter nodal location of Speaker 1. You can review node numbering by going to the “Mesh” tab. The editable field next to the “Nodes in Layer” button allows you to specify layer number you wish to inspect for nodes. Negative sign in front of the number indicates reversed phase.
2. **Speaker 2/3/4/5/6/7/8/9 Node** - Same as above. If you have only one speaker, you must enter ‘*’ (asterisk) in this field. Negative sign in front of the number indicates reversed phase.
3. **Mike Node** - This is an editable field and is used to enter nodal location of the Listener. Again, you should review numbering of nodes as described in bullet 1.

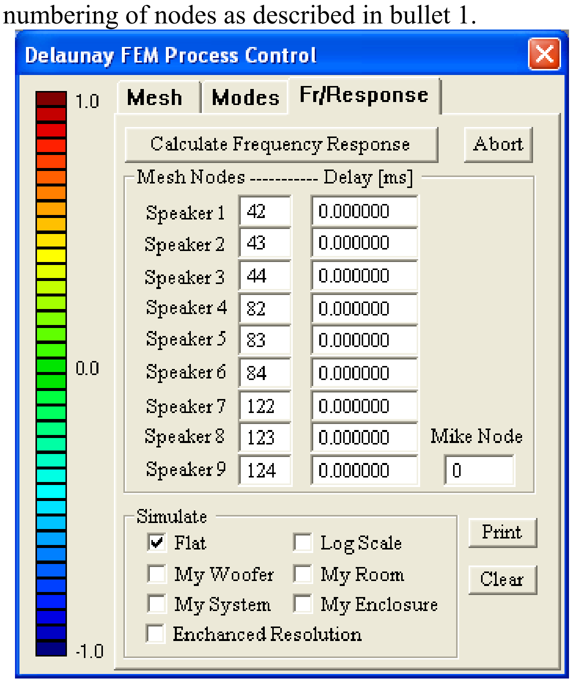


Fig 15.22 Delaunay Frequency response TAB

4. **Calculate Frequency Response** - Pressing this button will activate the algorithm. The process can be quite lengthy (2-3hours), particularly for larger number of nodes. Plotted frequency response represents acoustic pressure developed within the room as generated by up to 9 speakers.
5. **Simulate** group of options:
   **Flat** = No other components are included
   **My Room** = SPL in the room with last calculated T/S-type enclosure SPL superimposed.
   **My Woofer** = SPL in the room with woofer transfer function superimposed.
   **My System** = SPL in the room with complete system transfer function superimposed.
   **My Enclosure** = Select to calculate Driving Point Impedance of your speaker enclosure. <u>This option MUST be selected for FEM modal analysis of your loudspeaker enclosure.</u>
   **Log Scale** = Switch to logarithmic scale.
6. **Clear** - Simple clears the display.
7. **Print** - Button invokes printing dialogue box.
8. **Abort** – Simply aborts plotting.
9. **Delay** – You can add small delay to each loudspeaker source.

<u>Note: Frequency response range for FEM analysis is selected from Preferences screen same as Box range.</u>

The colored scale on the left-side of the dialogues box indicated normalized pressure at a node. Please note, that the display windows can be moved around and re-sized, however, the display is not scaled accordingly to the size of the window. Scaling the graphics will be incorporated in later releases of the program.

The process of creating a mesh, starting with the SPLG graph, has been described in some details already. You have a choice of using several mesh densities, by selecting the triangle size from the provided list box. You will notice, that triangle size is selected independently of the meshing grid size (the X / Y / Z element size). The grid resolution should reflect the size of the object being analysed. For instance, when analysing the acoustics of an open floor house, you may want to select X=0.25meter, Y=0.25 meter and so on. This will allow you to draw plastered walls between rooms with correct scale. After that, you can decide on the actual FEM mesh density, by selecting one of the available triangle sizes.

On the other hand, when you work with loudspeaker enclosure modal analysis, a better choice of grid would be X=0.01m or X=0.02m and so on.

Generally, large triangles will result in less dense mesh, and this will result in lower usable frequency of analysis. However, it will also give you much shorter calculation time. So we would recommend, that you start with less dense mesh and then increase the density, if you need the analysis progressing accurately into higher frequencies (above 200-1000Hz).

After the mesh has been created, you can review the location of individual layers / nodes by editing the layer number and pressing the **"Show Layer**" button. For this software release, there are 9 layers of nodes, numbered 0-9. Evidently, there are 8 layers of wedge elements.

The Delaunay triangulation (wedge element) FEM scheme is equipped with very similar functionality to the brick element FEM scheme. File Upload and Save functions are available via the main menu and the saving and loading process is a standard, Windows-type of control dialog. Once you are happy with the mesh density, it is always recommend to save the mesh into a file, for future reference.

One of the features of the **"Fr/Response"** TAB, allows you to automatically view the location of a node in the mesh, when you edit the node number in one of the nine "loudspeaker" data fields provided on this TAB. The layer with this particular node will be highlighted in the **"3D Mesh Window"**, so please keep it opened at the same time.

Changing the type (LOG – LIN) frequency scale in the SPL plotting window can be performed when the **"My Enclosure" checkbox in NOT selected**. Generally, it is recommended to make the scaling decision before plotting, and set the type of the scale then.

For instance, after you calculate the mesh ( after the Calculate Mesh button was pressed ), you can save the resulting mesh into a file and later load it back into the program – see main menu options above. This ensures, that exact same mesh will be used for all subsequent calculations. Figure 15.23 was created with re-loaded mesh.

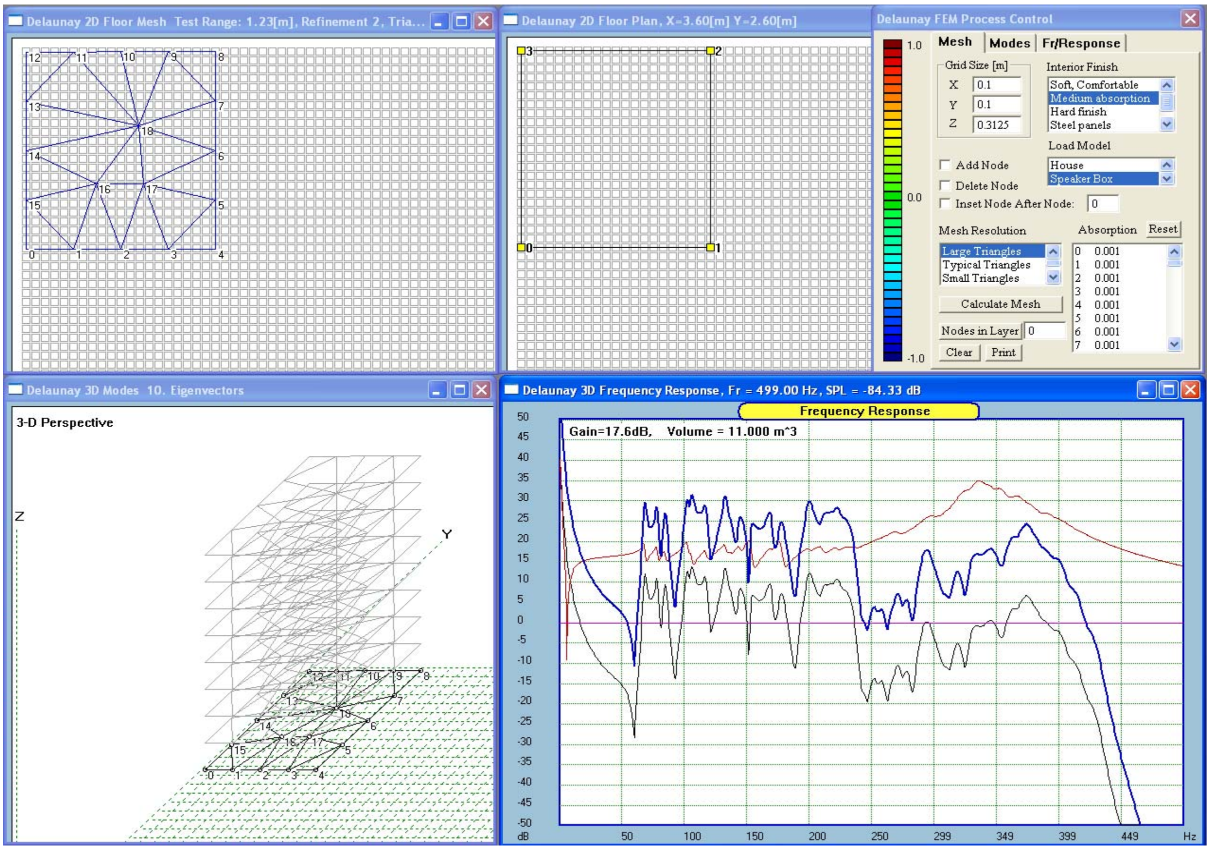



Fig 15.23 FEM SPL potting function.

It is easy to observe, that the SPL plot above is valid within ~200Hz range. This is due to the size of triangles used for mesh creation.

**Some aspects of subwoofer placement**

As we know, all rooms (a cavity volume enclosed by walls) have resonant frequencies at which the SPL generated by a source can be quite large. The frequencies at which resonances occur (called **modes**), depend on the geometry of the room. At a resonance frequency the pressure pattern inside the room will consist of **antinodes**, where the pressure is maximum, and **nodes**, where the pressure is zero. In a room with hard (eg. reflective) walls, the pressure will always be maximum at the wall or in a corner when the room is excited at any of its modal frequencies. If the source is located at an anti-node for a given mode, the room response will be greatest. If the source is located at a pressure node for a given mode, the room will not respond regardless how powerful the source is. Some of us (myself including) take advantage of our listening room's acoustic behavior to improve the performance of our systems. Placing a loudspeaker or subwoofer against a wall or in the corner of a room allows the low frequency modes of the room to enhance the low frequency performance of the loudspeakers. When the source is placed at a node for that frequency standing wave pattern, the maximum room response drops to zero at that frequency. Moving the speaker some distance away from the node restores back (at least partially) the room's response. Modal patterns only occur when the room is being driven at a modal frequency. At any other frequency, the pressure waves radiating outwards from the source reflect from the walls, but do not combine to produce a modal pressure pattern. As a result, there is no nodes and antinodes and the pressure can actually fall to zero at a wall. You will see these irregular patterns on Figure 15.17 – Figure 15.21.

**Eliminating Room Modes Using Multiple Subwoofers**

If a woofer is placed against a wall, it energizes all of the modes along the length of the room. If the woofer is placed in the corner (of rectangular room, as we saw before, in non-rectangular rooms this will not be the case in general) , it will energize all mores in this room. On one hand, we may want to have more low frequency output, but on the other, room modes have very definitely resonant character and will add excessive SPL at this frequency and will add ringing in time, as we find out in the next chapter. Understanding what happens in listening rooms at low frequencies requires determination of the room modes. In addition, we need to understand what happens if we start moving the source of the sound (loudspeaker) and the listener around the room. Finally, we need to take another look at the sound pressure level distribution when several loudspeakers are placed in the room. The floor plan of our example room is shown below.

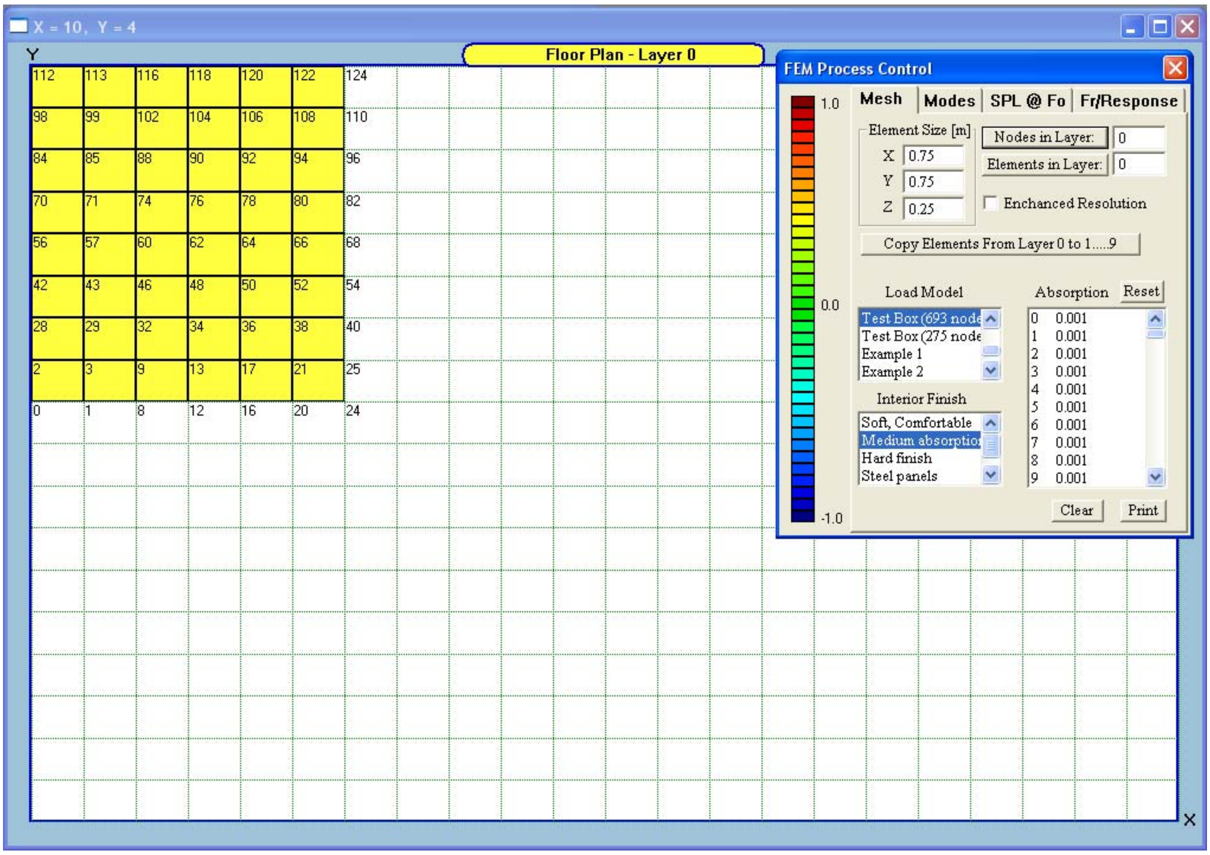



Figure 15.24 Floor plan of our test room

Calculating and displaying modal pressure pattern certainly helps the understand where the pressure peaks are located. In addition, a color scheme is used to indicate weather the pressure peak is maximum in positive polarity (red color on the chart) or maximum in the negative polarity (blue color on the chart). It is easy to observe, that the sound pressure on opposite side of a null in standing wave pattern are in opposite polarity. This gives us an excellent opportunity to place second subwoofer in the negative polarity area to cancel the positive maximum pattern generated by the first subwoofer.

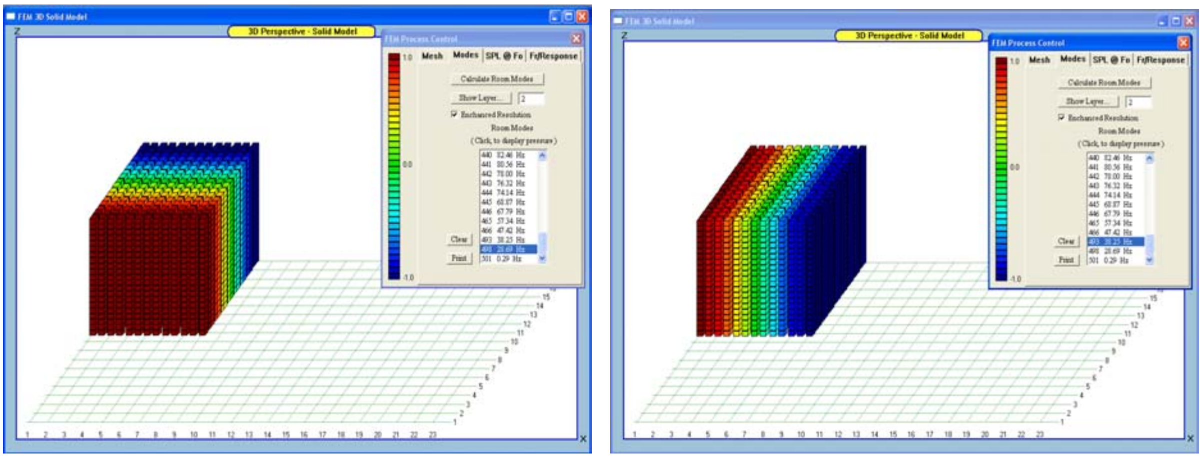



Figure 15.25. Modes: 28.7Hz and 38.3Hz

To visualize the modal problem in terms of the loudspeaker frequency response (SPL), we can start examining the SPL plot as taken with a simulated sealed enclosure with F3dB = 30Hz and placed in one of the corners – this would be corner 124 as marked on our floor plan. Nodal frequencies of 28.7Hz, 38.2Hz, 47.4Hz 57.3Hz 67.7Hz and so on, are clearly visible on the SPL plot as a series of sharp peaks – see Figure 15.21.

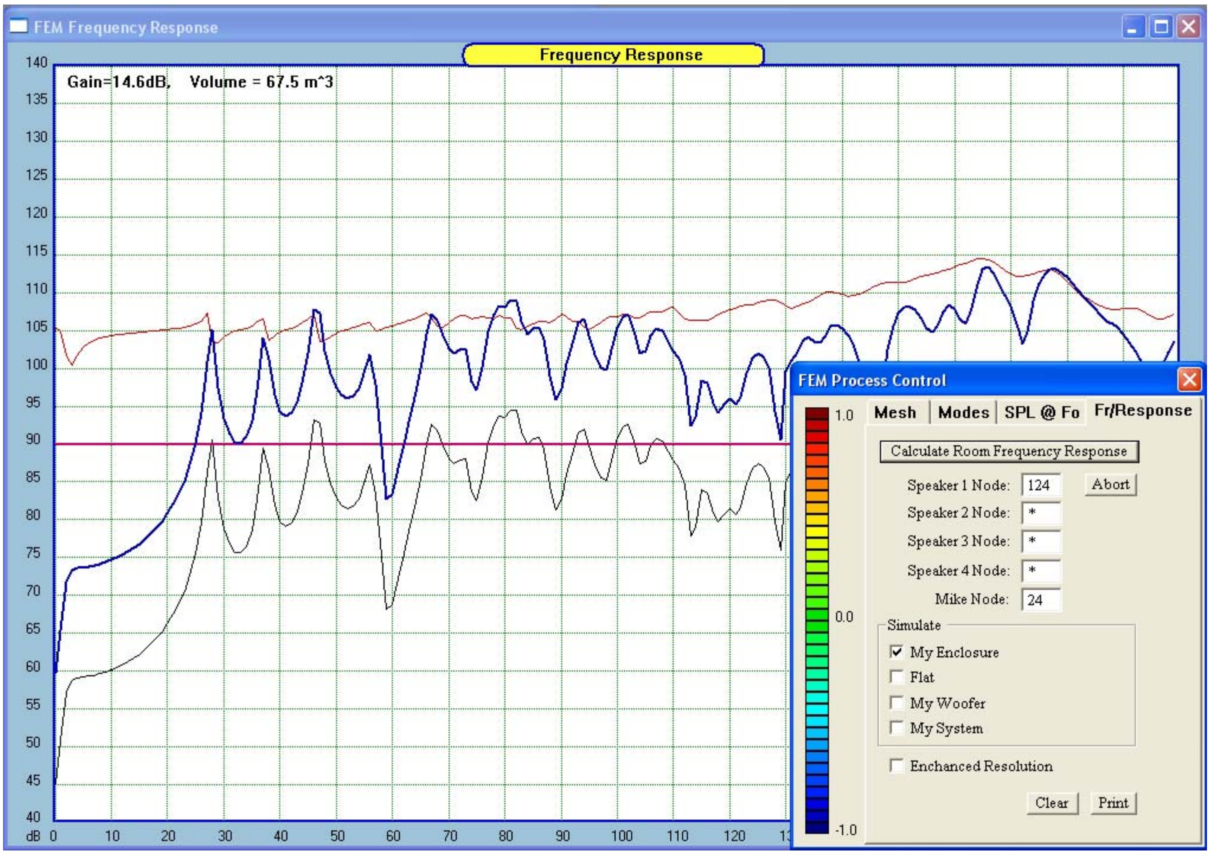


Figure 15.26. The room's modal influence is clearly seen on SPL curve – peaks at low frequencies.

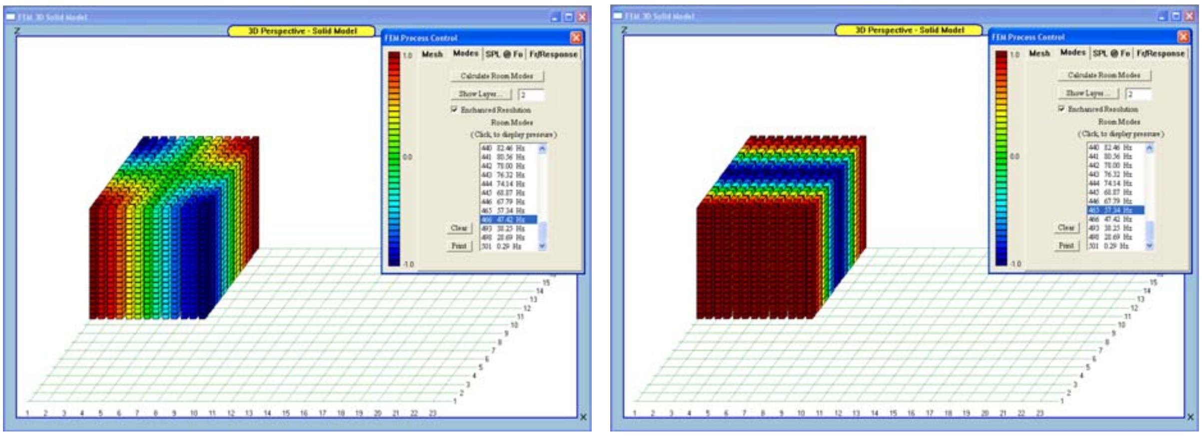


Figure 15.27. Modes: 47.4Hz and 57.3Hz

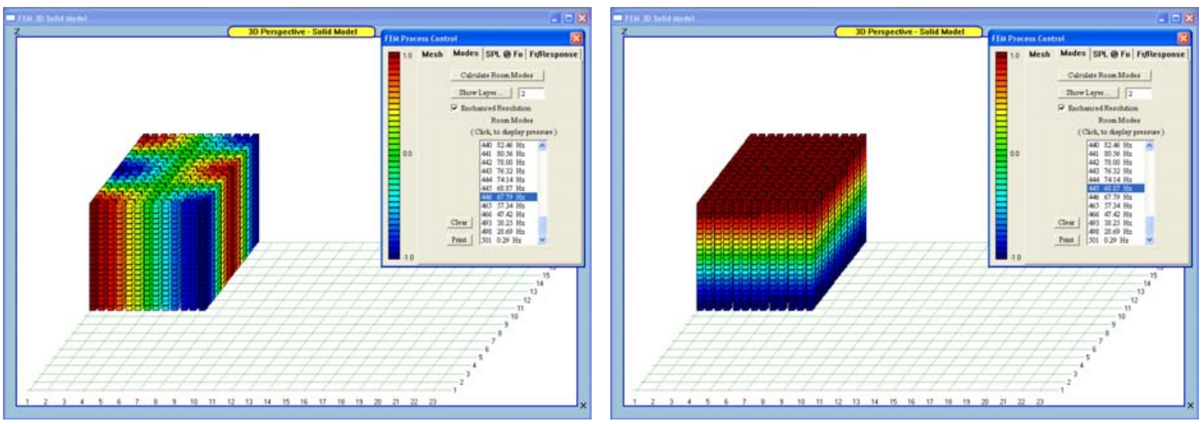


Figure 15.28. Modes: 67.8Hz and 68.7Hz

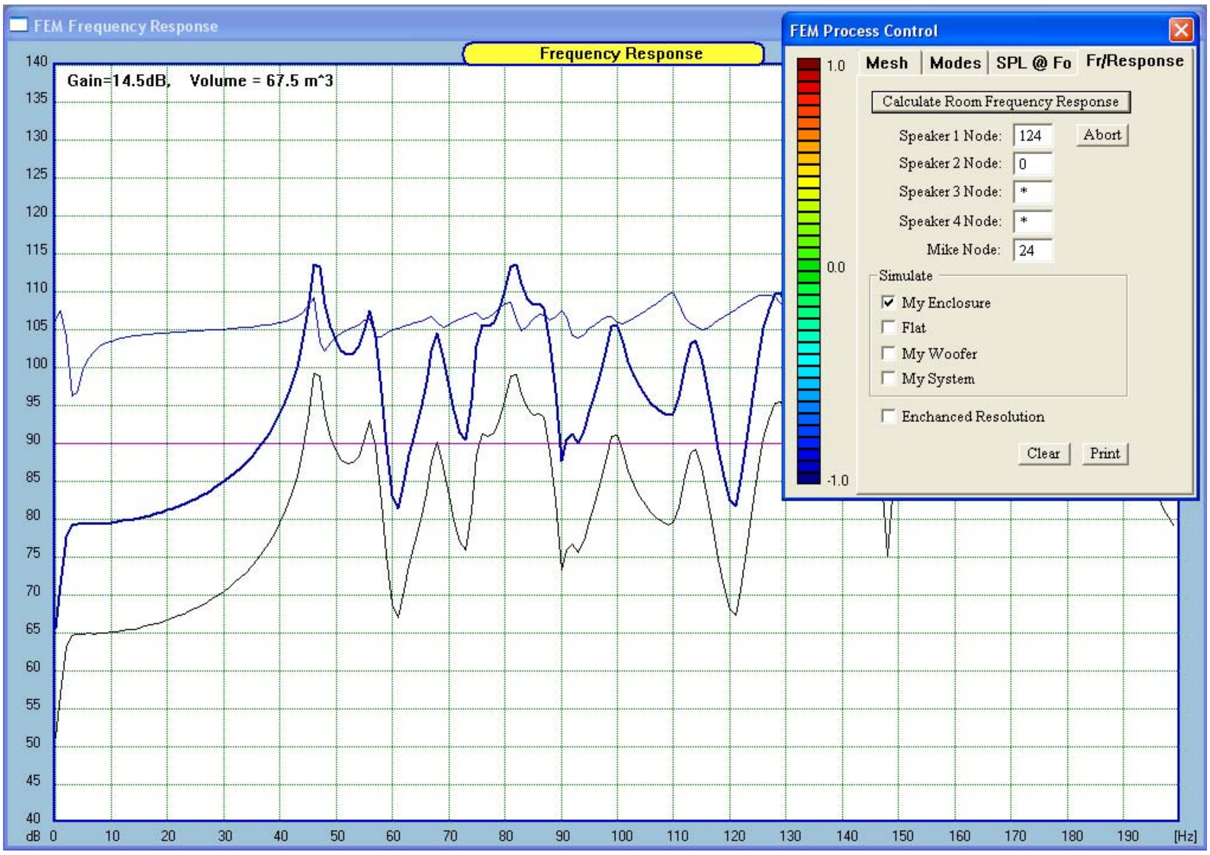



Figure 15.29 Two woofers at opposite corners

As we place more subwoofers in the room, we are able to cancel other low-frequency modes at 28.7Hz and 38.3Hz – see Figures 15.22 and 15.23. There is one **tangential** mode of 47Hz left. This mode generates the same polarity pressure at the opposite corners of the room and as such, our corner subwoofer placement would not eliminate this mode. You may need to use electronic equalization for this mode.

**There is another very positive thing happening in our room. If you examine the SPL below 25Hz, you will find, that it has been steadily increasing as we were canceling more nodes using additional woofers. This may be of interest to many readers, as it will be much easier to conceal 4 smaller subwoofers in the room instead of one really large device. At 10Hz (movie special effects) the gain is about 12dB for 4 subwoofers – see Figure 15.30.**

**Secondly, by distributing subwoofers, we have achieved a 12dB improvement in frequency response smoothness below 50Hz. Please compare Figure 15.26 ( 107 – 73dB = 34dB variations) and Figure 15.30 (107 – 85dB = 22dB variations). The SPL variation reduction can be further improved by adding more subwoofers.**

**Anything In-Between – Modal Equalization.**

Electronic filtering or equalization for the purpose of improving the perceived quality of sound reproduction via loudspeakers in a listening room has been in widespread use for a number of years now. Intelligent equalization can work, and is useful in improving the overall perceived quality in sound reproduction.

With the advent of currently available powerful capabilities of digital signal processing (DSP) technology and fast desk-top computers, some researchers have become interested in more complex methods of equalization. These methods include "room de-reverberation" or inverting the room transfer function for the purpose of using it as a filter. It is probably rather intuitive to realize, that equalization is **position-dependant**. Audio waves that add in one location will most certainly subtract in another location, or at least interact with differing amplitudes and phases. As the wavelengths are becoming shorter and shorter at higher frequencies, the listener position accuracy is becoming more critical. If taken to absurd level, this would require you to remain very still during your listening experience.

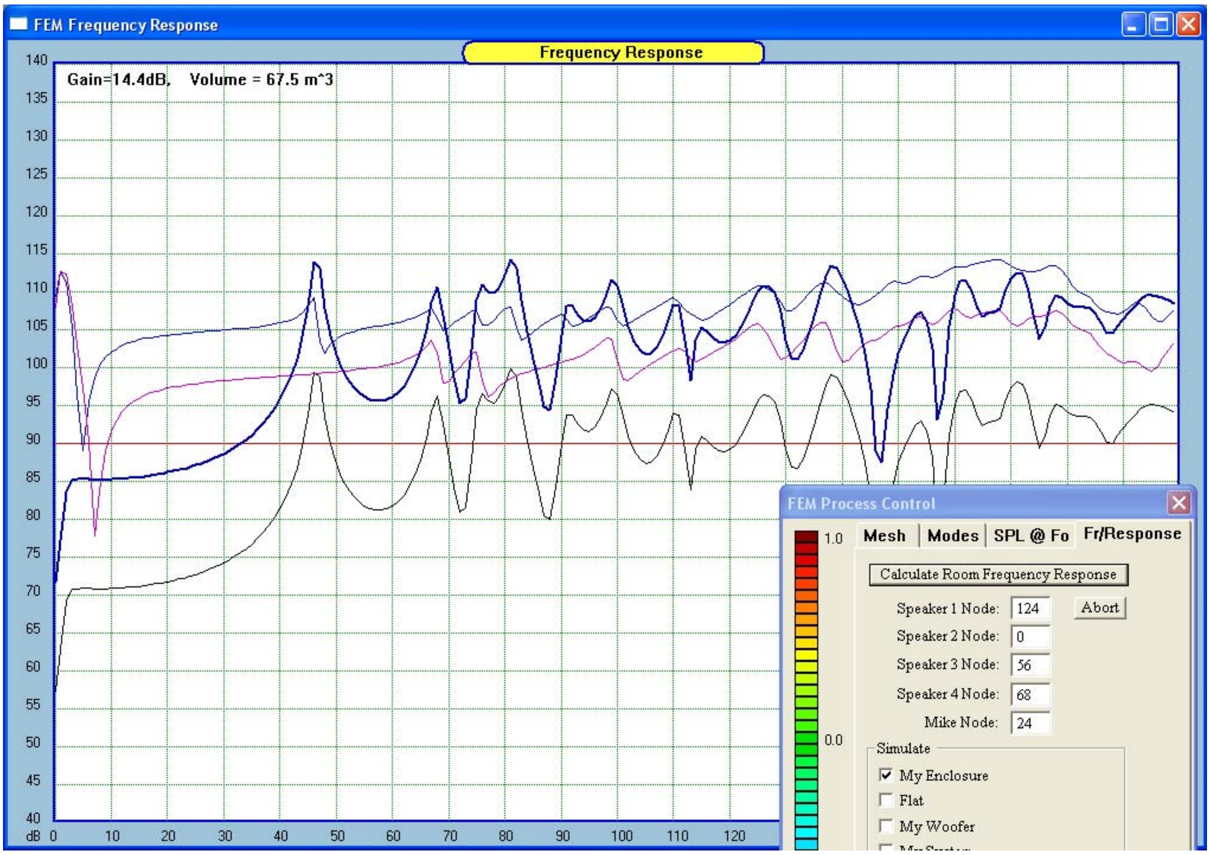


Figure 15.30. Four woofers: two at opposite corners and two half-way along the longest wall

Secondly, it should also be fairly obvious, that any attempt to equalize a dip, would require extra power. And a lots of it too. A 10 dB dip compensation requires 10 times more power. A 20 dB boost uses 100 times more power. Because “dip compensation” is no different than adding a sharp resonance to the frequency response, we have just added a 10 or 20 dB resonance that will be very disturbing at most locations other than the one we are attempting to equalize.

**It can not be stressed enough, that the key to successful room equalization is in understanding what can and cannot be corrected with the help of filters or DSPs.**

Room resonances, add artifacts to the sound. Some people refer to this as “room playing along with the musicians”. In the frequency domain, this effect would manifest itself as peaks and dips in the frequency response. In the time domain, room resonances manifest themselves as amplified and extended ringing of the impulse response. Acoustically, room resonances are those things that are referred to as “boomy bass”, “one-note-bass”, “muddy bass”, “ringing”, “coloration”. They make it difficult to follow bass notes.

Room resonances behave mostly as minimum-phase phenomena, therefore, they can be equalized with the following in mind.

1. Carefully measure and analyze what the problem is. Attempt to determine where the room modes are and what is their order. Computer modeling and predictions will offer many useful hints here.
2. Is it a peak, that you attempt to deal with or a dip ?.
3. Dips caused by sound-wave cancellations are impossible to equalize. Dips are very position sensitive.
4. Peaks are possible to deal with. Typical room will exhibit modal Q-factors of finite value, possibly in the vicinity of 5-20. This would indicate, that selective SPL output reduction at the offending room modes may actually flatten the frequency response to the acceptable level. This issue can be addressed with carefully matched parametric equalizers.

You may expect, that after peak equalization, the subsequent room measurements would return significantly shorter ringing of the impulse response and significantly smoother frequency response. Electronic, selective modal equalization ( Type I ) will be discussed at the end of this chapter.

**Absorption Coefficient At The Node**

It is often important to control absorption of some selected area of your listening room. You may have absorbers fixed on the walls or heavy carpet laid on some area of the room. This information needs to be available to the program for more accurate SPL predictions. The FEM Screen allows you to **assign the absorption coefficient to a single node**, anywhere within your model. The node number you wish to assign the absorption value must be known, so the right time to enter the coefficients would be after you created the model. You can use any of the built-in FEM models or edit your own and save it. Entering the coefficients is quite straightforward.

1. Examine node number you wish the Absorption Coefficient to be associated with. Node numbers for each layer can be displayed as explained before.
2. Knowing the node numbers, you can now DOUBLECLICK on the coefficient's list box to open a small data-entry dialogue for entering **Absorption Coefficients between 0.001 – 1.00 ( 0.1% - 100%).**

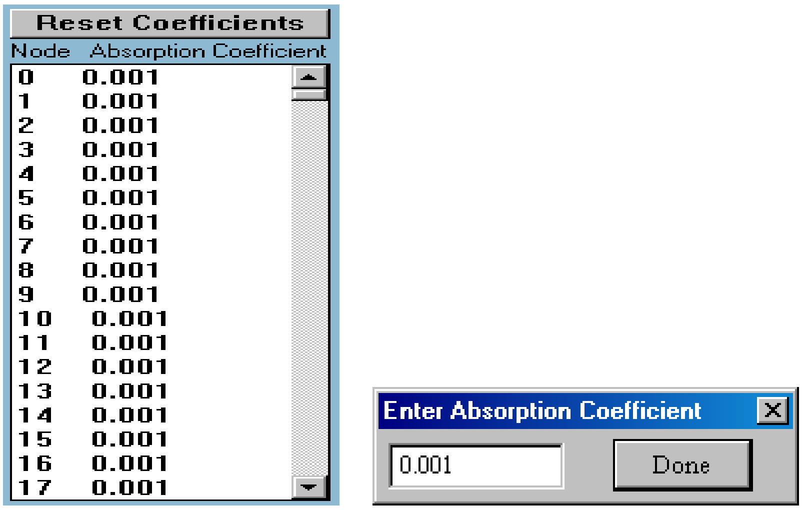


Figure 31. Entering Absorption coefficients.

Large areas with high absorption can dramatically change the pressure distribution within the room. Example on Figure 15.32 shows pressure distribution for the built-in "Test Box 1" at around 110Hz, where all nodes have uniformly identical absorption of 0.001. Figure 15.33 shows dramatic change after the front wall of the room was made totally almost absorbent ( AC = 0.99 ). Please note on the associated frequency response plot, that the "0Hz" pressure mode (always present in enclo0sed spaces) is no longer there for this case.

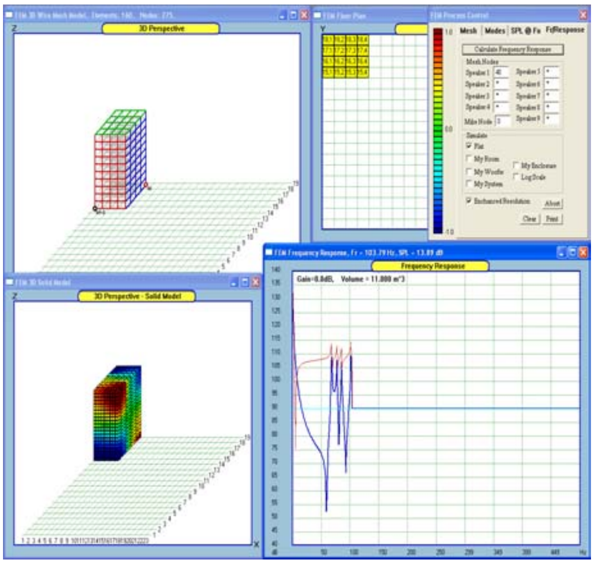


Figure 15.32. AC = 0.001 at all nodes

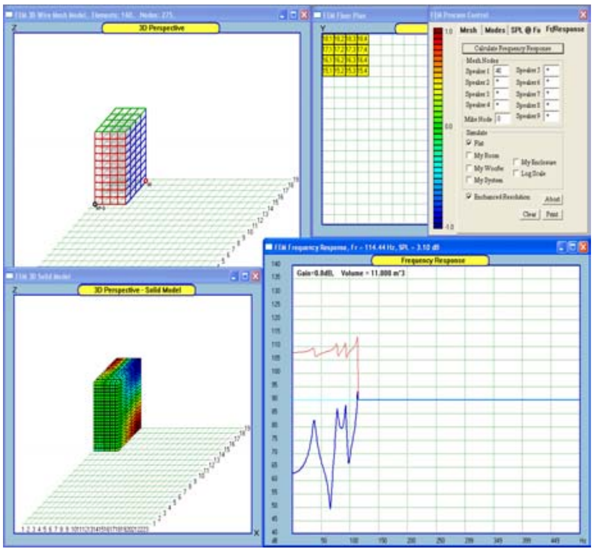


Figure 15.33 AC = 0.99 at the front wall.

It is observable on Figure 15.33, that the nodal pressure pattern for 79Hz node ( blue and red colour ) is confined to rear corners of the room. It does not extend beyond half –way towards the front of the room. If the room was closed, the blue and red pattern would continue to right to the front wall. The way we defined this example, practically "removes" the front wall from the model. The microphone is placed and Node 3, which is near the left-bottom corner and is right in front of the "missing" wall.

Whit the help of Absorption Coefficient, you will be able to approximate opened windows, carpets, heavy drapes and many other objects made from materials of known absorption. The absorbing area is determined by the location and the number of adjacent nodes with nominated Absorption Coefficients. As a refresher, in the next paragraph, we discuss some basic concepts of Absorption Coefficient.

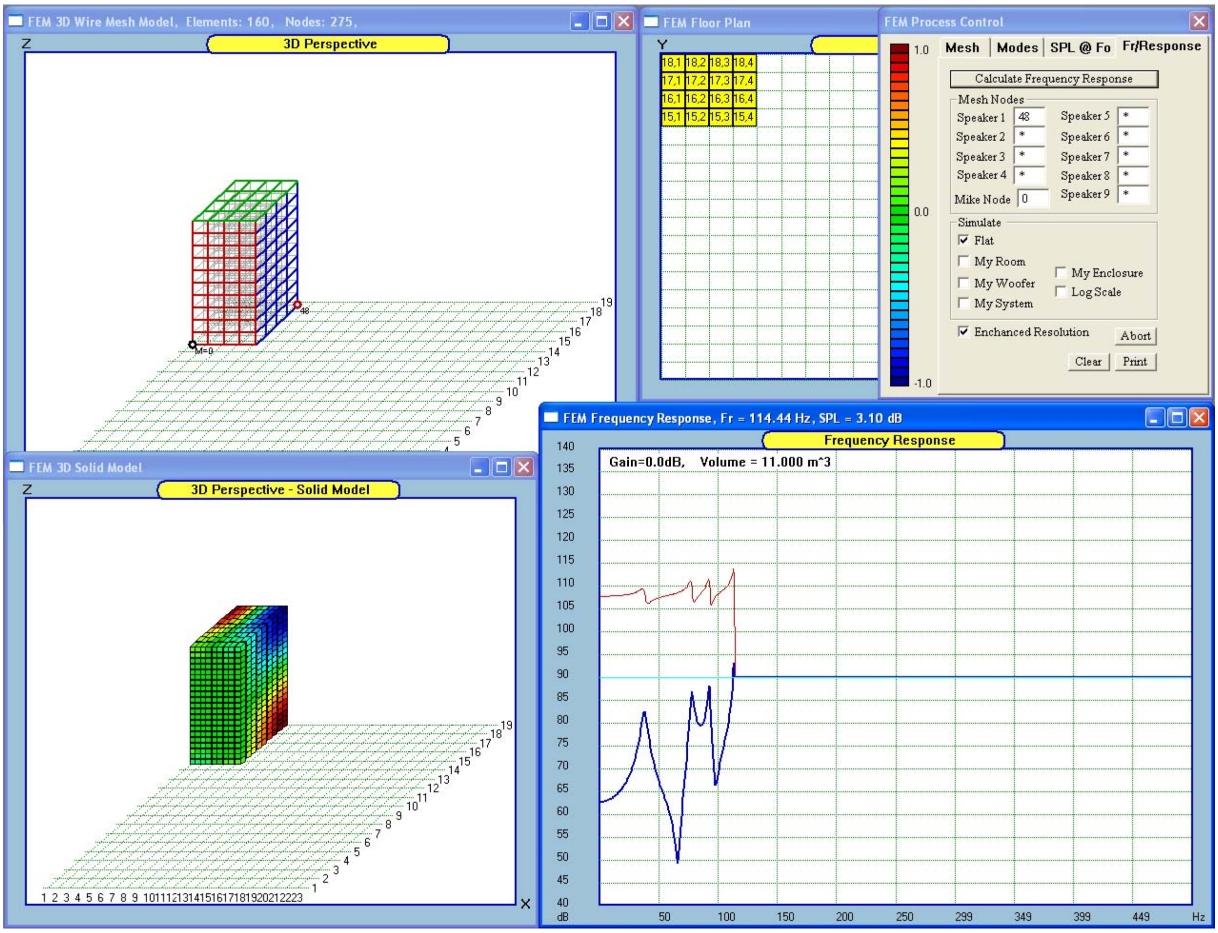


Figure 15.34. Front wall of the room is 100% absorbent (opened to the outside world ).

**Absorption**

Acoustic wave incident on a surface can be absorbed or reflected back. The amount of absorbed energy depends on physical properties of the surface material. We would normally define the fraction of the wave energy absorbed by the material as the **absorption coefficient "a"**.

**Typical values of a are between 0 and 1.** If our surface is composed of a material exhibiting absorption coefficient of 1 (for instance an open window), then all of the incident energy would be absorbed into the surface. An absorption coefficient of 0.4 means, that 40% of the incident energy would be absorbed into the surface and 60% is reflected back into the room.

The ability to absorb sound will vary with frequency for any given absorbing material. Typically, higher frequencies are better absorbed than lower frequencies because sound power of a typical audio spectrum diminishes with frequency, therefore the absorbing material does not need to works as hard.

There are some interesting materials listed in the absorption table. For instance, heavy folded curtains absorb nearly all sound energy at 4kHz and above. This would be a good choice for the back wall of the listening room if you need to suppress the rear reflections at those frequencies. Another material good for mid-range frequencies would be thick fiberglass, followed by acoustic paneling.

Placement and absorption characteristics of the material will help you modify the acoustical properties of your listening room. You can (1) control early reflection so some degree and (2) affect total reverberation time of the room.

The FEM screen models the SPL response for frequencies below 200Hz. We would recommend using the Absorption Coefficients from the first column (125Hz) from the table below.

Tables of absorption coefficients are published, usually stating the figure at six standard frequencies. Shown below are the coefficients of some common substances at these frequencies:

| **Material** | **125 Hz** | **250 Hz** | **500 Hz** | **1kHz** | **2 kHz** | **4 kHz** |
|---|---|---|---|---|---|---|
| | a | a | a | a | a | a |
| Concrete | 0.01 | 0.01 | 0.015 | 0.02 | 0.02 | 0.02 |
| Plasterboard | 0.29 | 0.1 | 0.05 | 0.04 | 0.07 | 0.09 |
| Acoustic panelling | 0.15 | 0.3 | 0.75 | 0.85 | 0.75 | 0.4 |
| Ac. Tiles | 0.1 | 0.35 | 0.7 | 0.75 | 0.65 | 0.5 |
| Brick | 0.024 | 0.025 | 0.03 | 0.04 | 0.05 | 0.07 |
| Carpet, thin | 0.05 | 0.1 | 0.2 | 0.25 | 0.3 | 0.35 |
| Carpet, thick,u/lay | 0.15 | 0.27 | 0.35 | 0.42 | 0.5 | 0.6 |
| Curtains | 0.05 | 0.12 | 0.15 | 0.27 | 0.37 | 0.5 |
| Curtains,heavy,folded | 0.2 | 0.35 | 0.5 | 0.65 | 0.8 | 0.95 |
| Fibreglass, 1 in | 0.07 | 0.23 | 0.42 | 0.77 | 0.73 | 0.7 |
| Fibreglass, 2 in | 0.19 | 0.59 | 0.79 | 0.92 | 0.82 | 0.78 |
| Fibreglass, 4 in | 0.38 | 0.89 | 0.96 | 0.98 | 0.81 | 0.87 |
| Floor,wood on joists | 0.15 | 0.2 | 0.1 | 0.1 | 0.1 | 0.1 |
| Floor,wood blocks | 0.05 | 0.05 | 0.05 | 0.07 | 0.1 | 0.1 |
| Glass | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.02 |
| Plaster on lathe | 0.3 | 0.2 | 0.1 | 0.07 | 0.04 | 0.03 |
| Plaster on brick | 0.024 | 0.027 | 0.03 | 0.037 | 0.039 | 0.034 |
| Plywood, 3/8 inch | 0.11 | 0.11 | 0.12 | 0.11 | 0.1 | 0.09 |
| Ply, 3/16 on 2 in batten | 0.35 | 0.25 | 0.2 | 0.15 | 0.05 | 0.05 |
| Wood, 3/4 in solid | 0.1 | 0.11 | 0.1 | 0.08 | 0.08 | 0.11 |

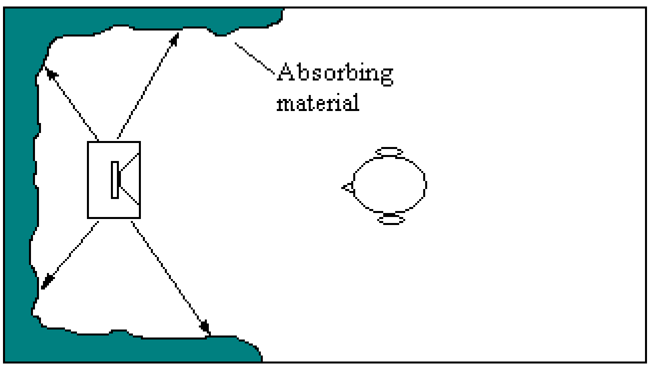



Figure 15.35 Absorbing material eliminates most of the early reflections from behind the speaker

In order to measure how much sound a particular surface material and area absorbs, we multiply the area (in square meters) by the absorption coefficient, to get the result in **Square Meter Absorption Units** or **smau** (the unit of sound absorption based on the square meter).

**Average Absorption Coefficient**

$$\bar{a} = \frac{a_1 s_1 + a_2 s_2 + \ldots\ldots a_n s_n}{S}$$

Where $a_1, a_2 \ldots a_n$ are the individual absorption coefficients of surfaces $s_1, s_2, \ldots s_n$.

**Modeling Real Sub-Woofer Placed in Room**

Evaluating real loudspeaker performance in-room can be accomplished by measuring or importing the "free-air" or anechoic frequency response of the loudspeaker and then using the FEM screen to create room transfer function. The "free-air" loudspeaker frequency response is automatically incorporated in the room transfer function if you check the **"My Woofer"** checkbox in the **"FEM Control – Frequency response"** dialogue box. Sub-woofers are modeled as a point sources located at the assigned nodes (Node 48 in the example below). You can place up to 4 sub-woofers in the room.

**Please remember, that the FEM screen uses linear frequency scale 1 – 200Hz.**

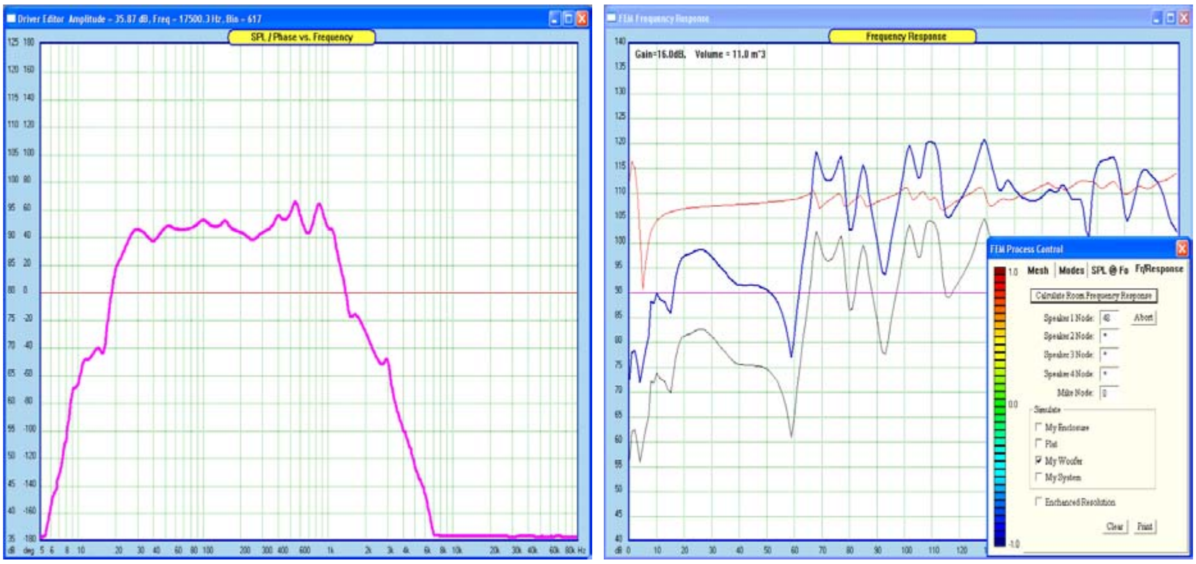


Figure 15.36. "Free air" and "in-room" frequency response of the same driver.

**Electronic Selective Modal Elimination**

As we mentioned before, loudspeaker-room system is dominated by the room properties for the rate of energy decay. At higher frequencies, typically above a few hundred Hertz, the energy decay is controlled to a required level selectively placing necessary amount of absorbing material in the room. This is generally adequate, as long as the wavelength of sound is small compared to dimensions of the room. At low frequencies passive means of controlling reverberant decay time become more difficult because the physical size of necessary absorbers increases and may become prohibitively large compared to the volume of the space, or absorbers have to be made narrow-band. Related to this, the cost of passive control of reverberant decay greatly increases at low frequencies. One option left is Electronic Modal Elimination using notch filtering approach. Example circuit is shown on Figure 15.29.

A6 output impedance (see Figure 15.29) is the used as one of the GYRATOR's components. Output is Node 4 and input is Node 0. Approximated value of the inductor, L, created with the gyrator:

**L = (R7 – Rout)*Rout*C6**

Rout = 100 ohm, output impedance of the A6. Rout should be selected from 100-470 ohm. Approximated value of the inductor's, Q, created with the gyrator:

$$Q = XL/(Rout+R4)$$

Methods for optimizing the response at a listening position by finding suitable locations for loudspeakers have been proposed but cannot fully solve the problem. Because of these reasons there has been an increasing interest in active methods of sound field control at low frequencies, where active control becomes feasible as the wavelengths become long and the sound field develops less diffuse.

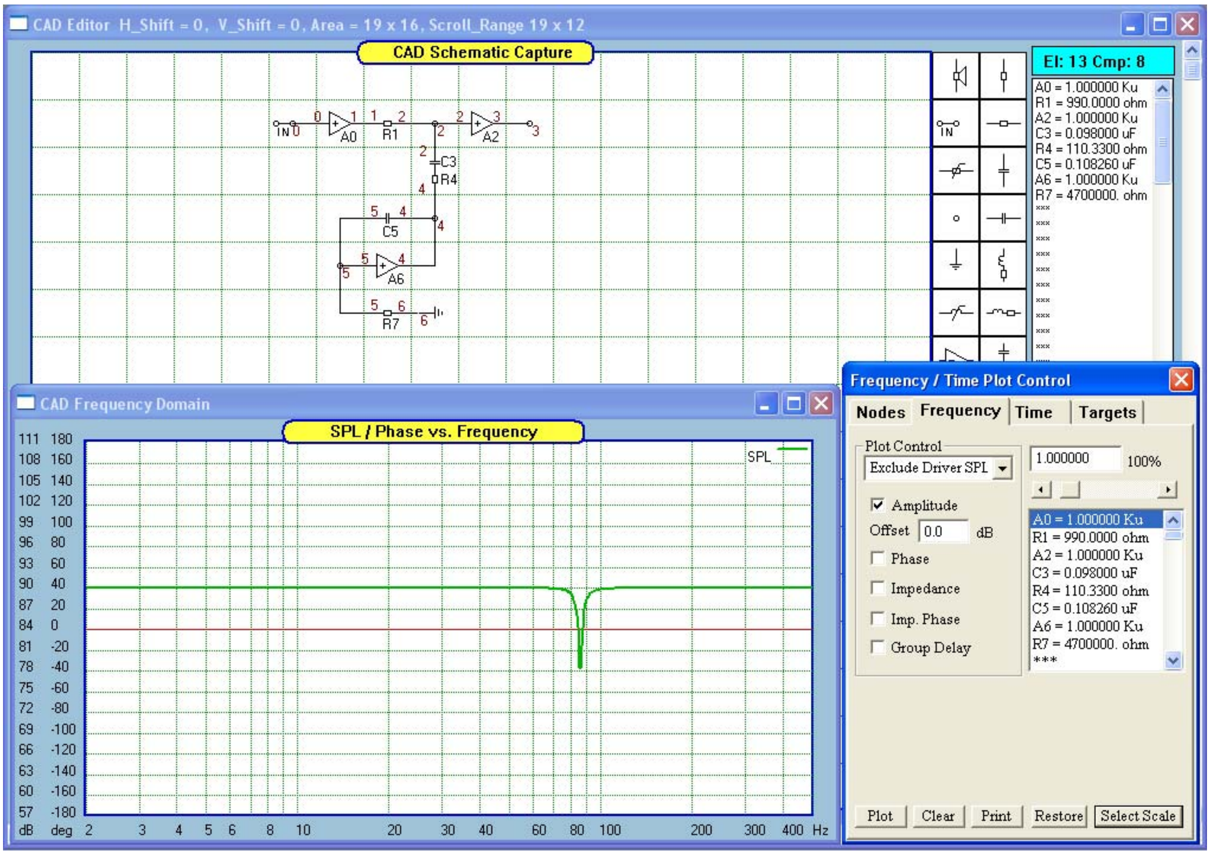



Figure 15.37. Electronic equalizer notch set to Fo = 87Hz.

Modal resonances in a room can be audible because they modify the magnitude response of the primary sound or, when the primary sound ends, because they are no longer masked by the primary sound. Detection of a modal resonance appears to be very dependent on the signal content. Olive et al. reports that low-Q resonances are more readily audible with continuous signals containing a broad frequency spectrum while high-Q resonances become more audible with transient discontinuous signals.

Traditional magnitude equalization attempts to achieve a flat frequency response at the listening location either for the steady state or early arriving sound. Both approaches achieve an improvement in audio quality for poor loudspeaker-room systems, but colorations of the reverberant sound field cannot be handled with traditional magnitude equalization. Colorations in the reverberant sound field produced by room modes deteriorate sound clarity and definition. Modal equalization is a novel approach that can specifically address problematic modal resonances, decreasing their Q-value and bringing the decay rate in line with other frequencies.

Modal equalization decreases the gain of modal resonances thereby affecting an amount of magnitude equalization. It is important to note that traditional magnitude equalization does not achieve modal equalization as a by-product. There is no guarantee that zeros in a traditional equalizer transfer function are placed correctly to achieve control of modal resonance decay time. In fact, this is rather improbable. A sensible aim for modal equalization is not to achieve either zero decay time or flat magnitude response. Modal equalization can be a good companion of traditional magnitude equalization.

A modal equalizer can take care of differences in the reverberation time while a traditional equalizer can then decrease frequency response deviations to achieve acceptable flatness of response. We have presented two different types of modal equalization approaches, Type I modifying the sound input into the room using the primary speakers. Type I systems are typically minimum phase. There are several reasons why modal equalization is particularly interesting at low frequencies. At low frequencies passive means to control decay rate by room absorption may become prohibitively expensive or fail because of constructional faults. Also, modal equalization becomes technically feasible at low frequencies where the wavelength of sound becomes large relative to room size and to objects in the room, and the sound field is no longer diffuse. Local control of the sound field at the main listening position becomes progressively easier under these conditions.

Type I system implements modal equalization by a filter in series with the main sound source, i.e. by modifying the sound input into the room. Modal equalization is a method to control reverberation in a room when conventional passive means are not possible, do not exist or would present a prohibitively high cost. Modal equalization is an interesting design option particularly for low-frequency room reverberation control.

Having designed  suitable notch filter using CAD screen, you **must plot its frequency response** to place the SPL data in the location accessible to FEM screen. To include the plot from CAD screen in FEM calculations, simply check the “My System” check box on the “FEM Control – Frequency response” dialogue box.

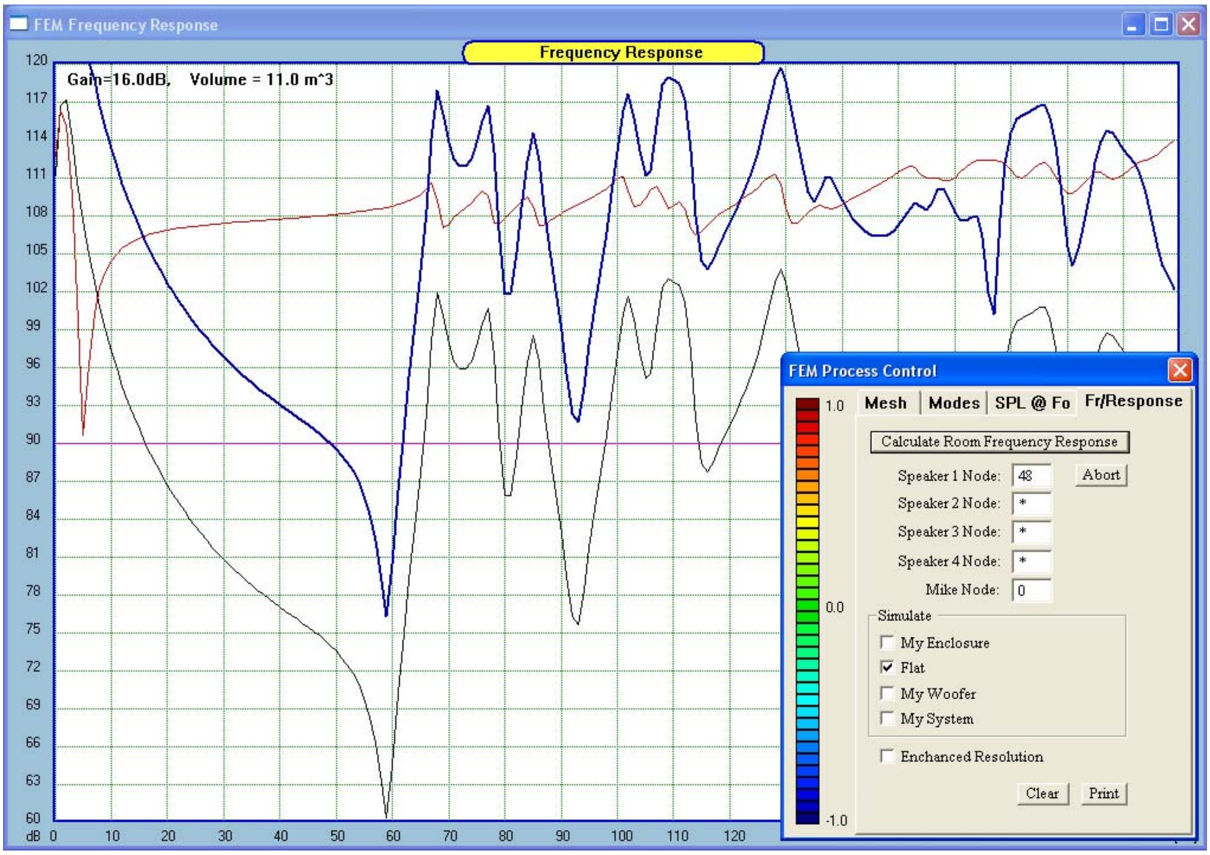


Figure 15.38. Room transfer function **before** EQ applied @ 87Hz.

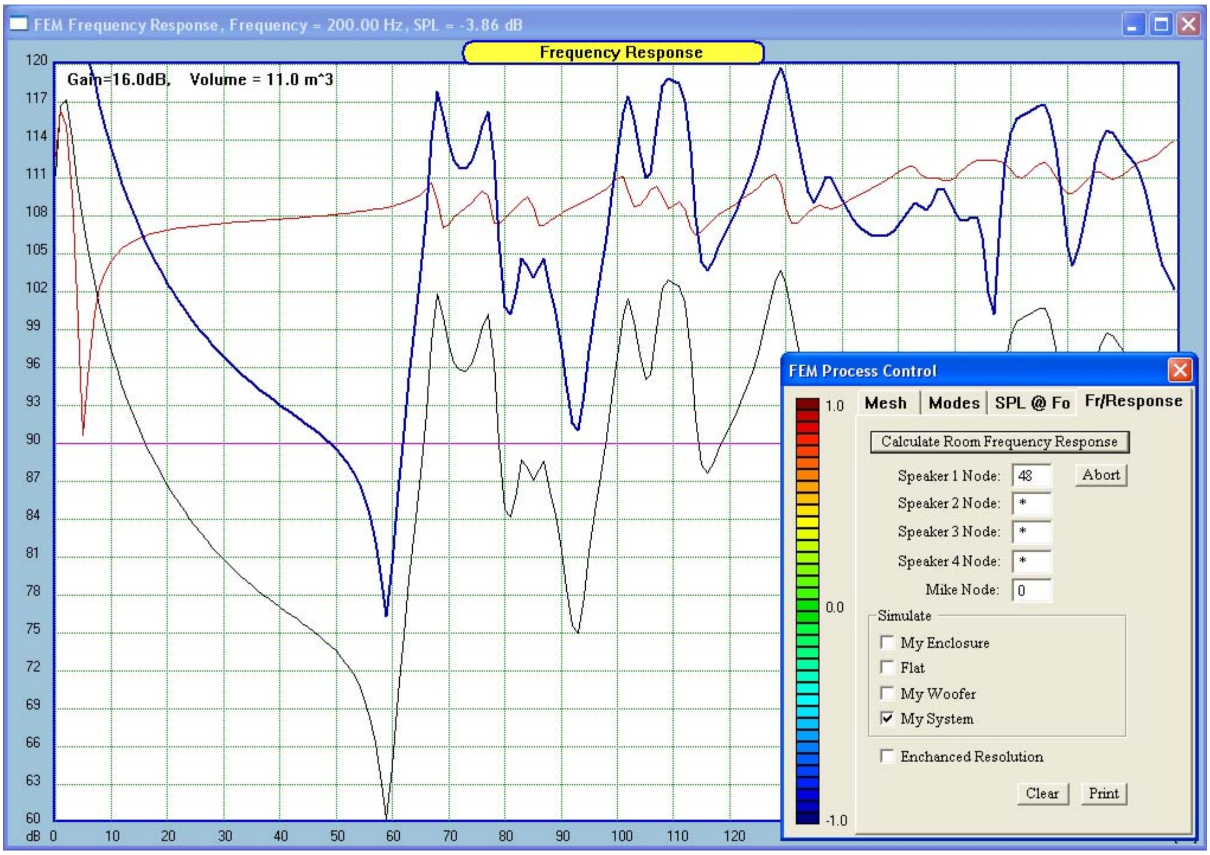



Figure 15.39. Room transfer function **after** EQ applied @ 87Hz.